Boombot: Low Friction Coefficient Stair Climbing Robot Using Rotating Boom and Weight Redistribution

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Abstract

Boombot comprising four wheels and a rotating boom in the middle has been developed as a mobile robot for stair climbing application. The boom can rotate continuously to provide flipping motion to the main body. Stair climbing with low friction co-efficient between the wheels and the ground has been implemented by weight distribution between the rear and front wheels. To arrive at the optimum weight distribution, static analysis of the Boombot over stairs has been carried out using linear programming. It is found that redistribution of weight results in static equilibrium with low friction co-efficient.

Keywords: Wheeled robot, Stair climbing, Friction, Linear programming

1 Introduction

Mobile robots have gained importance in recent years as an aid in low intensity conflict operations for the law enforcement agencies. These operations are carried out predominantly in urban scenarios where stair climbing ability is an essential feature in robots. Capability of wheeled robots has been enhanced by addition of mechanisms enabling stair climbing. One such mechanism is a continuously rotating boom which provides flipping motion to the robot. Example of such a robot is the *Pointman* developed by ARA-Robotic Systems [1].

Inspired by such robotic systems, CAIR has developed a Boombot. The robot performs well on stairs having rough surface. However, the robot fails to climb stairs made of polished granite stone without weight redistribution. In order to guarantee climbing, configuration of the robot needs to be determined which ensures static equilibrium in all states with low friction. During climbing the boom makes contact with the stairs at an angle which makes static analysis non-trivial. The static force balance equations result in an under-constrained system. In order to find numerical solution for the reaction forces at the points of contact additional physical constraints are given and the problem is posed as a linear optimization problem seeking solution which minimizes the maximum ratio of traction force to normal reaction. Analysis of various wheeled robots has been carried out using this methodology which include *Shrimp* robot [2]&[3] and *Sample Return Rover* of JPL [4]. Traction optimization for multi-wheeled robots has been developed using linear programming in [5]. The technique has been applied to simulation of stair climbing robot *ENSIETA* in [6] using addition of inverted pendulums. Similar method is used in [7] for simulation of

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force actuator based suspension vehicle. This methodology has also been extended to caterpillar locomotion based robots for stable posture analysis in [8].

Linear optimization gives feasible solutions which guarantee that all physical constraints are met. It is found that for stair climbing a feasible solution is possible with redistribution of weight. Though this solution results in slip-climb cycle during stair climbing, it does not require overall weight increase or special wheel material for increasing friction.

2 System Description

Boombot is a lightweight four wheeled robot developed for operations in urban scenarios. A rotating boom is provided at the middle of the robot to enable stair climbing. The step climbing sequence of the boombot using rotating boom is shown in Fig.(1).

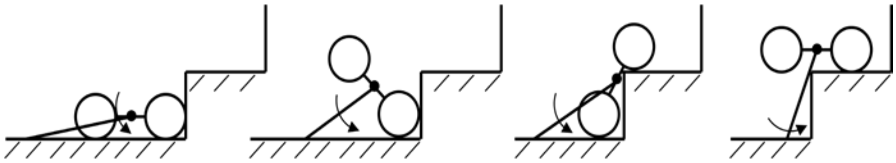


Figure 1: Sequence showing the step climbing of boombot using rotating boom

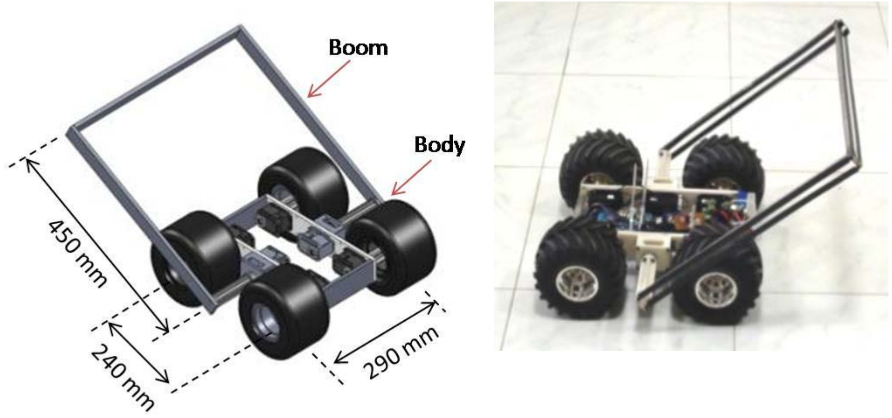


Figure 2: CAD model and physical model of Boombot

CAD model and the physical system developed is shown in Fig. (2). The robot comprises two sections: body and boom. The body consists of housing and wheels. Wheel base of the body is 240 mm and the wheel track is 290 mm. The housing contains fours motors for driving the wheels, two motors for driving the boom through an external gearbox of ratio 3:1, control and communication electronics and battery pack. All six motors are identical integrated servomotors capable of providing 1.5 Nm torque and peak speed of 52 rpm. Lithium polymer battery pack having 8Ah capacity is used for providing power. Total weight of the body is 3.8 Kgs of which 800 grams is contributed by battery pack. The wheels are 160 mm diameter semi-pneumatic rubber tires. The boom is a U-shaped acrylic frame weighing 400 grams and having a length of 450 mm.

3 Kinematic configuration

In order to climb stair case with run S_r and rise S_h the essential conditions for the wheel base $2L_w$ and the boom length L_2 are listed below:

- For climbing first step of staircase wheel base $2L_w > S_h$ as shown in Fig.(3a)
- For enabling flipping of the body over the first step of staircase the boom length $L_2 > (L_w + r + S_h)$ as shown in Fig.(3b) where, r is the radius of the wheel.

Staircase used for testing Boombot has S_r =290 mm and S_h =170 mm. The wheelbase of 240 mm and boom length of 450 mm meet the above mentioned conditions.

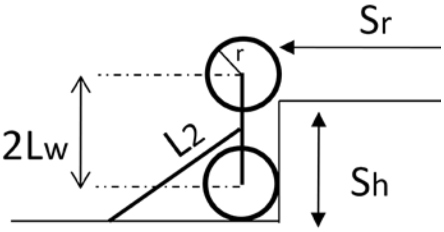


Figure 3a: Climbing first step

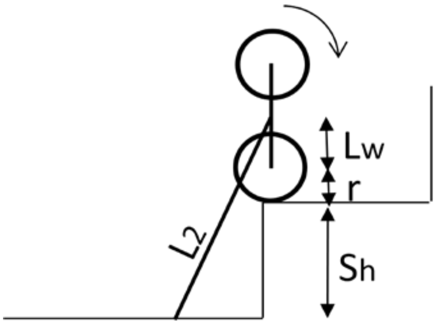


Figure 3b: Flipping over first step

4 Static Analysis

Static analysis has been carried out for staircase climbing considering redistribution of body weight which results in shifting of CG from the centre of the body. Dynamics effects have been ignored since the speed of Boombot is low during stair climbing. The analysis provides the information about the CG offset required to enable static equilibrium with low friction co-efficient between wheel and the ground. Fig. (4) shows the free body diagram of Boombot. The typical case where the front wheel is just on the edge of the step, the rear wheel is in air and the boom makes contact with the lower step has been taken for analysis. In this state the Boombot makes two contacts with stairs; one at front wheel P_1 and second at the boom P_2 at distance L_2' from the boom-body joint. N_1 and T_1 are the normal reaction and traction force at P_1, while N_2 and T_2 are the normal reaction and traction force at P_2. Boombot parameters required for analysis are given in Table 1. The angle between body and the boom, θ, is used as the variable.

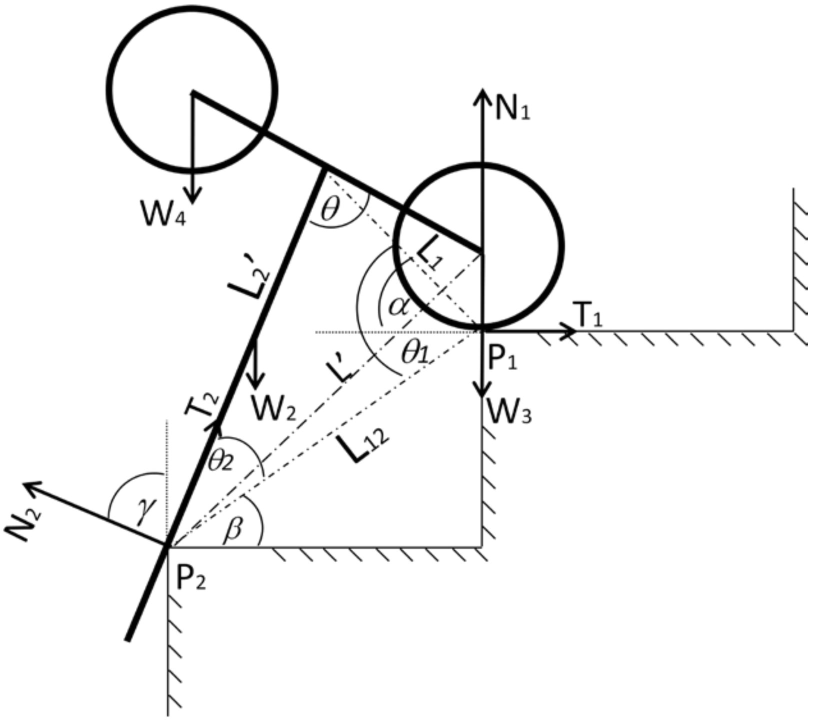


Figure 4: Free Body Diagram of the Boombot on staircase

Table 1: Parameters used for analysis:

Parameter	*Symbol*	*Value*
Wheelbase	$2L_w$	0.24 m
Boom Length	L_2	0.45 m
radius of wheel	r	0.08 m
Mass of the body	W_1	3.8 kg
Mass of boom	W_2	0.4 kg

The analysis has been carried out with redistribution of the body weight W_1, resulting in effective weight W_3 on front wheel and W_4 on the rear wheel.

Thus

$$W_3 = K\,W_1 \tag{1}$$

$$W_4 = (1 - K)\,W_1 \tag{2}$$

where, K is the weight redistribution factor. For the case where CG of the body lies at the centre, K = 0.5.

In order to carry out the static analysis for the given state depicted in Fig. (4), the length L_2' and the angle the boom makes with the step, γ, need to be determined. Using geometry these can be calculated as shown below,

$$L_{12} = \sqrt{{S_h}^2 + {S_r}^2} \tag{3}$$

$$\beta = \tan^{-1} \frac{S_h}{S_r} \tag{4}$$

$$L' = \sqrt{{L_{12}}^2 + r^2 - 2L_{12}r \cos(90+\beta)} \tag{5}$$

${L_2}'$ can be determined from the quadratic Eq. (6) and taking the positive value

$${L_2}'^2 - 2\,{L_2}' L_w \cos\theta + {L_w}^2 - L'^2 = 0 \tag{6}$$

$$\theta_2 = \sin^{-1}\left(\frac{L_w \sin\theta}{L'}\right) + \sin^{-1}\left(\frac{r\sin(90+\beta)}{L'}\right) \tag{7}$$

$$L_1 = \sqrt{{L_{12}}^2 + {L'_2}^2 - 2L_{12}L'_2\, cos\,\theta_2} \tag{8}$$

$$\theta_1 = \cos^{-1}\left(\frac{{L_1}^2 + {L_{12}}^2 - {L'_2}^2}{2L_1L_{12}}\right) \tag{9}$$

$$\alpha = \theta_1 - \beta \tag{10}$$

$$\gamma = \theta_2 + \beta \tag{11}$$

The static equilibrium condition gives equation

$$\boldsymbol{AX} = \boldsymbol{B} \tag{12}$$

where

$$\boldsymbol{X} = \begin{bmatrix} N_1 \\ T_1 \\ N_2 \\ T_2 \end{bmatrix},\ \boldsymbol{A} = \begin{bmatrix} 1 & 0 & \cos\gamma & \sin\gamma \\ 0 & 1 & -\sin\gamma & \cos\gamma \\ S_r & -S_h & 0 & 0 \end{bmatrix}$$

and

$$\boldsymbol{B} = \begin{bmatrix} W_2 + W_3 + W_4 \\ 0 \\ W_3S_r + W_4(S_r - 2L_1\cos\alpha) + W_2\left(L'_2 - \frac{L_2}{2}\right)\cos\gamma \end{bmatrix}$$

There are four unknowns in Eq. (12) and only three equations. Thus no unique solution is possible. To find a solution, the problem is formulated as a linear programming minimization problem by providing additional physical constraints. No-slip condition can be guaranteed if the ratio of traction force and normal reaction at wheel contact is less than the corresponding friction co-efficient. In order to arrive at this solution, minimization of the traction force is taken as the objective function. The problem can be stated as:

Minimize the objective function

$$f = T_1 \tag{13}$$

For the equality constraint

$$\boldsymbol{AX} = \boldsymbol{B}$$

With inequality constraints on Ti's so that they are less than corresponding $\mu_i N_i$ (i=1,2)

$$\|T_1\| < \mu_1 N_1,\ \|T_2\| < \mu_2 N_2 \tag{14}$$

where, μ_1 : static friction coefficient between wheel and ground = 0.34

μ_2 : static friction coefficient between boom and ground = 0.2
have been determined experimentally.

Defining lower bounds on N_i's

$$N_1 \geq 0,\ N_2 \geq 0 \tag{15}$$

And lower and upper bounds on Ti's to be within the maximum torque of the wheel motors, $\tau_{w\,max}$ (2 x 1.5 Nm) i.e.,

$$-\frac{\tau_{w\,max}}{r} \leq T_i \leq \frac{\tau_{w\,max}}{r} \tag{16}$$

With the above problem definition, solution is found using simplex method in MATLAB. Once, normal reactions and traction forces have been determined the reaction torque required at the boom-body joint can be calculated by applying moment balance equation:

$$\tau = -L_2{}' N_2 + \left(\frac{L_2}{2}\cos\gamma\right) W_2 \tag{17}$$

5 Results

The normal reaction and the traction at the contact points P_1 and P_2 are found for different values of angle between the body and the boom (θ) for a given value of weight distribution factor (K). The peak value of the ratio T_1/N_1 is found for various values of K (0.5 to 0.7) and shown in Fig. (5).

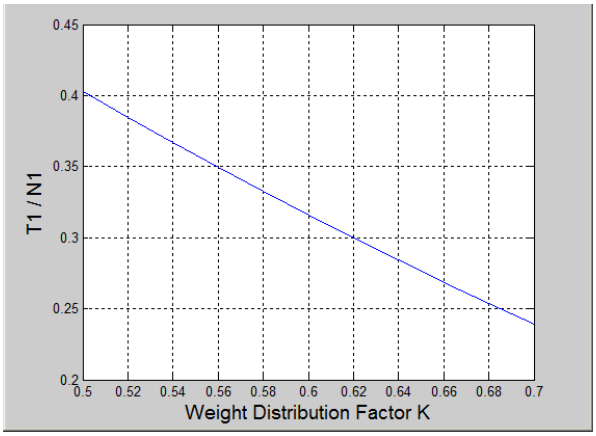


Figure 5: Plot of weight distribution factor (K) Vs maximum ratio T_1/N_1

It can be seen that for stairs having μ_1 less than 0.32 the weight redistribution factor K should be greater than 0.6. This value results in redistribution of Boombot body weight of 3.8 Kg such that the front wheel supports 2.28 Kg and rear wheel supports 1.52 Kg. Shifting the batteries, weighing 0.8 Kg, from center to front portion of the body results in the redistribution of weight such that 1.5 Kg acts on rear wheel and 2.3 Kg acts on front wheel.

Fig. (6a) and Fig. (6b) show the plots of the ratios T_1/N_1 and boom motor torque τ , respectively, for various values of body-boom angle θ at values of K equal to 0.4, 0.5 and 0.6.

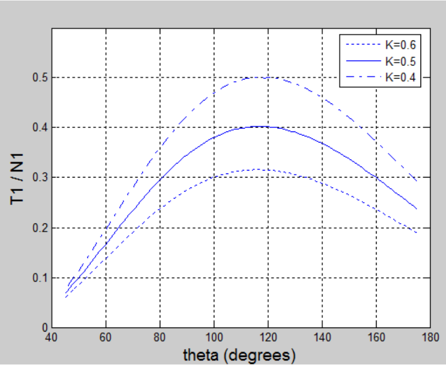


Figure 6a: Plot of ratio T_1/N_1 Vs body boom angle (θ) for K=0.4, 0.5 and 0.6

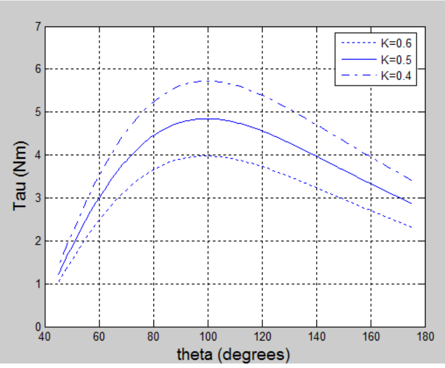
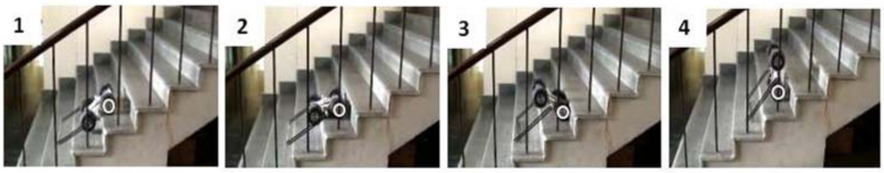


Figure 6b: Plot of ratio boom motor torque τ (tau) Vs body boom angle (θ) for K=0.4, 0.5 and 0.6

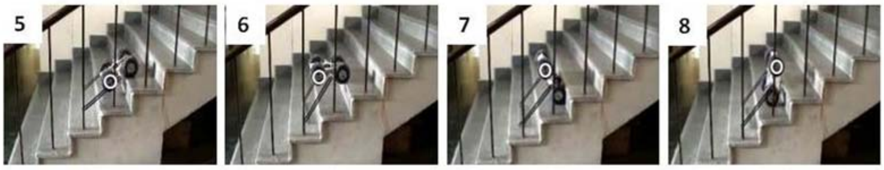
It can be seen from Fig. (6a) and Fig. (6b) that the maximum friction co-efficient required at the wheel ground contact and the torque required by the boom motor to maintain equilibrium increases as the weight redistribution factor is decreased. K equal to 0.6 corresponds to the state where wheel supporting higher weight is in front and in contact with step. In this state the wheel is able to pull the Boombot up to the edge of next step. The boom rotation then brings the rear wheel, which supports lower weight, in contact with the next step. This condition corresponds to weight redistribution factor K equal to 0.4. As seen from the Fig. (6a) friction co-efficient required for equilibrium is 0.5 which is less than the actual value of 0.34. Hence the wheel slips back to the previous step. The rotation of the boom again brings the wheel supporting higher weight in contact with the step and the Boombot continues to climb.

6 Conclusion

A mobile robot with rotating boom has been built for stair climbing application. It is shown that by redistribution of weight between the front and rear wheels the robot is able to climb stairs having low friction co-efficient. Weight redistribution results in higher normal reaction at wheel ground interface. Thus higher traction force can be applied by the wheel. Linear programming has been used to determine the ground reaction forces and optimal weight distribution to achieve static equilibrium. It is shown that a weight redistribution factor of 0.6 enables step climbing with friction co-efficient as low as 0.32. Weight redistribution has been implemented on the Boombot for successful stair climbing. Fig. (7) shows the snapshots of the Boombot climbing stairs.



Boombot climbing step with wheel supporting higher weight (marked white) in front



Boombot slipping over step with wheel supporting lower weight in front

Boombot climbing next step with wheel supporting higher weight having come to the front

Figure 7: Snapshots of Boombot climbing stairs in CAIR campus. The sequence shows the climb-slip-climb cycle needed for climbing stairs.

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