Stability Analysis of a Spinning and Precessing Viscoelastic Rotor Model Under the Effect of Tensile Centrifugal Force

S. Bose, A. Nandi, S. Neogy

Abstract

The present work deals with stability analysis of a spinning viscoelastic rotor mounted on a rotating (precessing) base under the effect of axial centrifugal tension, where the spin axis and precession axis intersect at right angle. The nutation speed is zero, the spin and precession speeds are considered to be uniform and the precession axis is located at one end of the shaft. The axial centrifugal force on the disc due to precession speed has been considered. The properties of the shaft material correspond to those of a linear viscoelastic model of four element type. The shaft-disc system is assumed to be axially and torsionally stiff. For analysis, a simple supported rotor has been considered with a rigid disc on a massless viscoelastic shaft. The governing 3rd order parametric equations for such a rotor are derived in the simultaneously spinning and precessing frame by using the principle of virtual work. The stability borderlines are computed using the generalized eigenvalue problem considering spin speed, precession speed and the centrifugal force on the disc as parameters. Variation in axial centrifugal tension is effected by varying the location of the disc from the precession axis on the shaft.

Keywords: Precession, Spin, Viscoelastic , Centrifugal tensile force

1 Introduction

Rotor on maneuvering base has presently become a topic of considerable interest and were studied by Lin and Meng[1], Das et al[2,3]. Passive damping technology using viscoelastic materials is traditionally used to control vibrations of structures [4]. The growing use of such structures has motivated many authors to study stability analysis of viscoelastic spinning rotor models [5].

It is known that a simultaneously spinning and precessing elastic rotor becomes unstable beyond a certain precession speed as demonstrated by Ghosh et al. [6,7]. Bose [8] considered the effect of stiffening of such rotors due to centrifugal force.

In the present work, the authors have investigated the stability of a rigid disc on viscoelastic shaft, which is simultaneously spinning and precessing under the effect of centrifugal axial force on the disc. A material model obtained by combination of

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elastic and viscous elements has been used in the analysis as formulated by Bland et al [9,10]. The stability borderlines are computed considering spin and precession speeds and the centrifugal force on the disc as parameters.

2 Formulation

A spinning viscoelastic shaft-disc system mounted on a precessing base is shown in Fig.1a. The objective of the present analysis is to find out the effect of spin and precession speeds on the stability of a rotor under the effect of axial centrifugal force on the rigid disc.

A rotor with a shaft and disc is shown in Fig.(1a). The rotor is spinning with a constant angular velocity Ω_s about an axis x. This axis is again precessing about the inertial Z axis with an angular velocity Ω_p. The inertial reference is represented by the coordinate system XYZ. The coordinate system $x'y'z'$ precesses about the axis Z with a uniform angular velocity Ω_p as shown in Fig.(1b). The reference xyz has an angular velocity of $(\Omega_p \hat{k} + \Omega_s \hat{i})$ where $\hat{k}$ is the unit vector along Z and $\hat{i}$ is the unit vector along the direction x. With respect to the reference xyz the shaft-disc system undergoes small deformations (Fig.2 & Fig.3). The symbols u_y, u_z, B_y, B_z represent the displacements and rotations of the disc along y and z directions respectively.

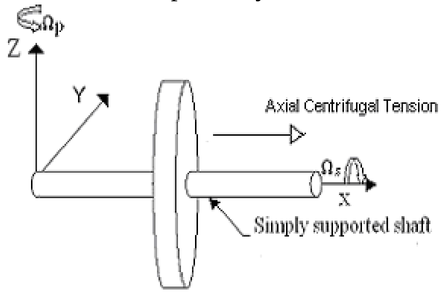

Figure 1a: A spinning and precessing rotor

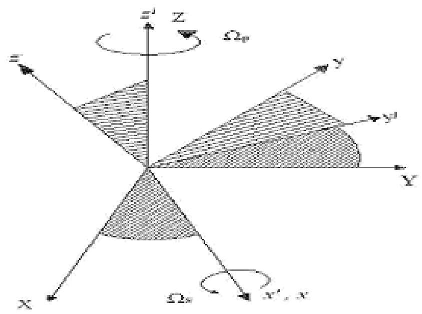

Figure 1b: Coordinate systems fixed to inertial and rotating references

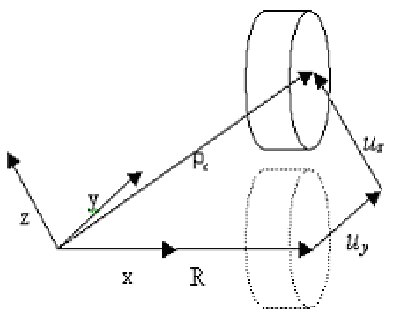

Figure.2:Location of deformed center of the disc.

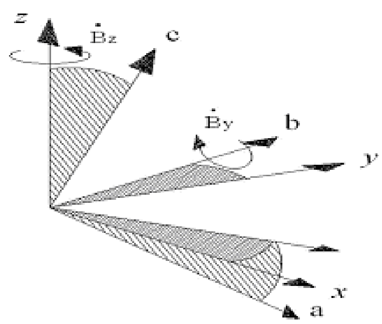

Figure.3: Rotations of the disc.

2.1 The variational formulation

The disc has a kinetic energy for its rigid body motion and deformation. The viscoelastic forces on the disc act as restoring forces. Axial centrifugal force on the rigid disc is also considered in the formulation. The Hamilton's principle for the shaft-disc system can be expressed as

$$\int_{t_1}^{t_2}\left(\delta T+\delta W_{ve}-\delta V_{ax}\right)dt=0 \tag{1}$$

Where, δT is the first variation of kinetic energy, δW_{ve} is the virtual work done by the viscoelastic forces and δV_{ax} is the first variation of potential energy due to axial centrifugal force. The kinetic energy is a function of the displacements and their first order time-derivatives.

$$T=T\left(u_i,\frac{\partial u_i}{\partial t}\right), i=1,2,....,n \tag{2}$$

Where, the symbol u_i stands for the displacement of the i_{th} degree of freedom.

The virtual work done by the viscoelastic forces is $\{\delta U\}^T\{F\}_{ve}$. The first variation of potential energy can be expressed as $\{\delta U\}^T\{F\}_{ax}$. The symbols $\{U\},\{F_{ve}\},\{F_{ax}\}$ stand for displacement vector , the viscoelastic force and the axial force respectively in the spinning and precessing reference xyz

Equation (1) reduces to $\int_{t_1}^{t_2}\left(\delta T+\{\delta U\}^T\{F\}_{ve}+\{\delta U\}^T\{F\}_{ax}\right)dt=0$

$$\int_{t_1}^{t_2}\left(\sum_i\frac{\partial T}{\partial\dot u_i}\delta\dot u_i+\sum_i\frac{\partial T}{\partial u_i}\delta u_i+\{\delta U\}^T\{F\}_{ve}+\{\delta U\}^T\{F\}_{ax}\right)dt=0 \tag{3}$$

After certain manipulations one can show,

$$\{\delta U\}^T\{F\}_{in}+\{\delta U\}^T\{F\}_{ve}+\{\delta U\}^T\{F\}_{ax}=0 \tag{4}$$

Where,

$$\{F\}_{in}=-\left\lfloor\frac{d}{dt}\left(\frac{\partial T}{\partial\dot u_1}\right)\quad\frac{d}{dt}\left(\frac{\partial T}{\partial\dot u_2}\right)\quad..\quad\frac{d}{dt}\left(\frac{\partial T}{\partial\dot u_n}\right)\right\rfloor^T+\left\lfloor\frac{\partial T}{\partial u_1}\quad\frac{\partial T}{\partial u_2}\quad..\quad\frac{\partial T}{\partial u_n}\right\rfloor$$

The above equation states that in the spinning and precessing coordinate system the sum of the virtual work done by the inertia forces, the viscoelastic forces and the axial centrifugal force is zero.

The equilibrium equations are as below:

$$\{F\}_{in}+\{F\}_{ve}+\{F\}_{ax}=\{0\} \tag{5}$$

2.2 Four degree of freedom model

The four degree of freedom model considers the small rotational deformations of the disc in addition to translational movements. In undeformed condition, the vector $R\hat{i}'$ is the center the disc with respect to $x'y'z'$ reference. In deformed state, the center has position vector and velocity in the same reference denoted respectively [6] as :

$$\bar{\rho}_c = R\hat{i}' + u_{y'}\hat{j}' + u_{z'}\hat{k}' \text{ and } \frac{d\bar{\rho}_c}{dt} = -\Omega_p u_{y'}\hat{i}' + \left(\frac{\partial u_{y'}}{\partial t} + \Omega_p R\right)\hat{j}' + \frac{\partial u_{z'}}{\partial t}\hat{k}' \quad (6)$$

Where, the deformations at the location of the disc along the directions y' and z' are denoted by $u_{y'}$ and $u_{z'}$ respectively.

The translational kinetic energy can be expressed as

$$T_{trans} = \frac{1}{2}\begin{Bmatrix} -\Omega_p u_{y'} \\ \left(\frac{\partial u_{y'}}{\partial t} + \Omega_p R\right) \\ \frac{\partial u_{z'}}{\partial t} \end{Bmatrix}^T \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} -\Omega_p u_{y'} \\ \left(\frac{\partial u_{y'}}{\partial t} + \Omega_p R\right) \\ \frac{\partial u_{z'}}{\partial t} \end{Bmatrix} \quad (7)$$

The angular location of the disc can now be expressed by the consecutive rotations as shown in Fig.(1b) and Fig.(3).
The rotational deformations of the disc can be described as two infinitesimal rotations $B_{z'}$ and $B_{y'}$ about the axes z' and b (close to y') respectively.

Now, retaining all the terms to avoid missing out of any resulting significant term in the process, the components of the angular velocity vector and rotational kinetic energy of the disc can be expressed in the coordinate system abc as follows [6]: -

$$\begin{Bmatrix} \omega_a \\ \omega_b \\ \omega_c \end{Bmatrix} = \begin{Bmatrix} -\left(\Omega_p + \dot{B}_z\right)\sin B_y + \Omega_s \\ \dot{B}_y \\ \left(\Omega_p + \dot{B}_z\right)\cos B_y \end{Bmatrix} \text{and } T_{rot} = \frac{1}{2}\begin{Bmatrix} \omega_a \\ \omega_b \\ \omega_c \end{Bmatrix}^T \begin{bmatrix} I_P & 0 & 0 \\ 0 & I_T & 0 \\ 0 & 0 & I_T \end{bmatrix} \begin{Bmatrix} \omega_a \\ \omega_b \\ \omega_c \end{Bmatrix} \quad (8)$$

Where, the polar and transverse mass moments of inertia of the disc are represented by the symbols I_P and I_T respectively. For thin discs $I_P = 2I_T$.
The inertia force now can be computed by considering appropriate terms in the Lagrange's equations from Eq.(4).
The stress-strain relation for a viscoelastic material of the shaft can be represented by a four element model (Fig.4) as described below:

$$\sigma = \bar{E}\varepsilon \quad \text{where } \bar{E} = \frac{\beta + \gamma\frac{d}{dt} + \psi\frac{d^2}{dt^2}}{1 + \alpha\frac{d}{dt}} \quad (9)$$

and various coefficients are obtained from [4]. For a thin beam for bending in the $x-z$ plane, $M_y = -\bar{E}I\frac{d^2 w_z(x)}{dx^2}$. Similarly in the $x-y$ plane, $M_z = \bar{E}I\frac{d^2 w_y(x)}{dx^2}$

Where M_y and M_z are moments about y and z axes respectively. The stiffness matrix can now be derived by using the first principle using Eq. (4) where, k_{ij} is defined as force at the $i-th$ degree of freedom for a unit displacement along $j-th$ degree of freedom keeping all other degrees of freedom (other than $j-th$) fixed.

$$\begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{ve} = -\begin{bmatrix} k_{11} & 0 & 0 & k_{14} \\ 0 & k_{22} & k_{23} & 0 \\ 0 & k_{32} & k_{33} & 0 \\ k_{41} & 0 & 0 & k_{44} \end{bmatrix} \begin{Bmatrix} u_y \\ u_z \\ B_y \\ B_z \end{Bmatrix} = -\frac{\overline{E}I}{l^3} \begin{bmatrix} k'_{11} & 0 & 0 & k'_{14} \\ 0 & k'_{22} & k'_{23} & 0 \\ 0 & k'_{32} & k'_{33} & 0 \\ k'_{41} & 0 & 0 & k'_{44} \end{bmatrix} \begin{Bmatrix} u_y \\ u_z \\ B_y \\ B_z \end{Bmatrix} \tag{10}$$

The viscoelastic stiffness matrix is obtained simply by replacing the modulus of elasticity E in an elastic stiffness matrix by the operator $\overline{E}$.

The shaft-disc system is under the effect of axial centrifugal force on the disc due to precession. The potential energy due to axial force is expressed as:

$$V_{AX} = \frac{1}{2} \begin{Bmatrix} \frac{dw_y}{ds} \\ \frac{dw_z}{ds} \end{Bmatrix}^T \begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix} \begin{Bmatrix} \frac{dw_y}{ds} \\ \frac{dw_z}{ds} \end{Bmatrix} ds \tag{11}$$

The disc is located at a distance a from the precession axis. Distance of the disc from the other end is b .The total length of the shaft is $l = a + b$

<u>For both end fixed beam:</u>

From first principles w_y and w_z can be expressed in terms of the four degrees of freedom. Using Eq.(11) and appropriate terms in Lagrange's equation, following relations are obtained for 2 different regions of length – from 0 to 'a' and the other from 'a' to 'l':

$$\begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{AX1} = F_1 \begin{bmatrix} \frac{6}{5a} & 0 & 0 & -\frac{1}{10} \\ 0 & \frac{6}{5a} & \frac{1}{10} & 0 \\ 0 & \frac{1}{10} & \frac{2a}{15} & 0 \\ -\frac{1}{10} & 0 & 0 & \frac{2a}{15} \end{bmatrix} \begin{Bmatrix} u_y \\ u_z \\ B_y \\ B_z \end{Bmatrix} \text{ and } \begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{AX2} = F_2 \begin{bmatrix} \frac{6}{5b} & 0 & 0 & \frac{1}{10} \\ 0 & \frac{6}{5b} & -\frac{1}{10} & 0 \\ 0 & -\frac{1}{10} & \frac{2b}{15} & 0 \\ \frac{1}{10} & 0 & 0 & \frac{2b}{15} \end{bmatrix} \begin{Bmatrix} u_y \\ u_z \\ B_y \\ B_z \end{Bmatrix} \tag{12}$$

where F_1 and F_2 are the axial forces acting on shaft segments 'a' and 'b' respectively.

<u>For simple supported beam:</u>

Similarly for simply supported beam

$$\begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{AX1} = F_1 \begin{bmatrix} \frac{6}{5a} & 0 & 0 & -\frac{1}{5} \\ 0 & \frac{6}{5a} & \frac{1}{5} & 0 \\ 0 & \frac{1}{5} & \frac{a}{5} & 0 \\ -\frac{1}{5} & 0 & 0 & \frac{a}{5} \end{bmatrix} \begin{Bmatrix} u_y \\ u_z \\ B_y \\ B_z \end{Bmatrix} \text{ and } \begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{AX2} = F_2 \begin{bmatrix} \frac{6}{5b} & 0 & 0 & \frac{1}{5} \\ 0 & \frac{6}{5b} & -\frac{1}{5} & 0 \\ 0 & -\frac{1}{5} & \frac{b}{5} & 0 \\ \frac{1}{5} & 0 & 0 & \frac{b}{5} \end{bmatrix} \begin{Bmatrix} u_y \\ u_z \\ B_y \\ B_z \end{Bmatrix} \tag{13}$$

Dynamic equilibrium equation for the shaft-disc system for 4 degree of freedom is:

$$\begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{in} + \begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{ve} + \begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{AX1} + \begin{Bmatrix} F_y \\ F_z \\ M_y \\ M_z \end{Bmatrix}_{AX2} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{14}$$

The governing equation for 4 degree freedom system is of the following form:

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + ([K] + [K_1] + [K_{AX1}] + [K_{AX2}] + [K_C]\cos 2\Omega_s t + [K_S]\sin 2\Omega_s t)\{U\} = \{0\} \tag{15}$$

$$[K] = \frac{\overline{E}I}{l^3}\begin{bmatrix} k'_{11} & 0 & 0 & k_{14} \\ 0 & k'_{22} & k'_{23} & 0 \\ 0 & k'_{32} & k'_{33} & 0 \\ k'_{41} & 0 & 0 & k'_{44} \end{bmatrix} \tag{16}$$

If the operator $\overline{E}$ is replaced by modulus of elasticity E, the elastic stiffness matrix is obtained.

2.3 Stability analysis

Considering the differential operator at the denominator of the stress-strain relation, Eq.(15) becomes a 3rd order equation with periodic coefficients. This equation is converted to first order equations, thrice in number.
Governing equation for the 4 degree freedom system is:

$$[M]\{\ddot{U}\}+[C]\{\dot{U}\}+(\lfloor K \rfloor+[K_1]+[K_{AX1}]+[K_{AX2}]+[K_C]\cos 2\Omega_S t+[K_S]\sin 2\Omega_S t)\{U\}=\{0\}$$

By substituting matrix $\lfloor K \rfloor$ from Eq.(16) and $\overline{E}$ from Eq.(9) in the above governing equation following equation is obtained:

$$\begin{aligned} &\alpha[M]\{\dddot{U}\}+([M]+\alpha[C]+\psi[K])\{\ddot{U}\}+([C]+\gamma[K]+\alpha[K_1]+\alpha[K_{AX}])\{\dot{U}\}+\alpha[K_C]\cos 2\Omega_S t\{\dot{U}\} \\ &+\alpha[K_S]\sin 2\Omega_S t\{\dot{U}\}+(\beta[K]+[K_1]+[K_{AX}])\{U\}+([K_C]+\alpha 2\Omega_S[K_S])\cos 2\Omega_S t\{U\} \\ &+([K_S]-\alpha 2\Omega_S[K_C])\sin 2\Omega_S t\{U\}=\{0\} \end{aligned} \tag{17}$$

Arranging the terms in state space matrix form:

$$\begin{bmatrix} \alpha[M] & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\begin{Bmatrix} \dddot{U} \\ \ddot{U} \\ \dot{U} \end{Bmatrix}+\begin{bmatrix} ([M]+\alpha[C]+\psi[K]) & ([C]+\gamma[K]+\varepsilon[K_1]+[K_{AX}]) & (\beta[K]+[K_1]+[K_{AX}]) \\ -I & 0 & 0 \\ 0 & -I & 0 \end{bmatrix}\begin{Bmatrix} \ddot{U} \\ \dot{U} \\ U \end{Bmatrix}$$

$$+\begin{bmatrix} 0 & \alpha[K_C] & [K_C]+2\alpha\Omega_S[K_S] \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{U} \\ \dot{U} \\ U \end{Bmatrix}\cos 2\Omega_S t+\begin{bmatrix} 0 & \alpha[K_S] & [K_S]-2\alpha\Omega_S[K_C] \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{U} \\ \dot{U} \\ U \end{Bmatrix}\sin 2\Omega_S t=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\text{Or } [\overline{A}]\{\dot{X}\}+[\overline{B}]\{X\}+[\overline{C}]\{X\}Cos 2\Omega_S t+[\overline{D}]\{X\}Sin 2\Omega_S t=0 \tag{18}$$

Where $\{X\}=\begin{Bmatrix} \ddot{U} \\ \dot{U} \\ U \end{Bmatrix}$

The assumed solution for kth element in the vector $\{X\}$ is expressed as

$$X^k=e^{\lambda t}\left(a_0^k+a_1^k\cos\Omega_s t+b_1^k\sin\Omega_s t+a_2^k\cos 2\Omega_s t+b_2^k\sin 2\Omega_s t+.......\right) \tag{19}$$

Proceeding in a systematic way followed by Nandi et al [11] , one finally obtains the generalized eigenvalue problem from which the stability analysis is performed.

$$[A]\{X\}=\lambda[B]\{X\} \tag{20}$$

Where, $[A]=([M]\otimes[D]_1+[K]\otimes[I]+[K_c]\otimes[P_c]+[K_s]\otimes[P_s])$ and $[B]=-[M]\otimes[I]$

The symbol $\otimes$ stands for Kronecker product. Kronecker product between two matrices is explained below: -

$$[P]\otimes[Q]=\begin{bmatrix} p_{11}[Q] & p_{12}[Q] & - & - \\ p_{21}[Q] & p_{22}[Q] & - & - \\ - & - & - & - \\ - & - & - & \end{bmatrix} \quad (21)$$

The matrices $[D]_1$, $[P_c]$ and $[P_s]$ are the first order differentiation matrix, cosine product operation matrix and sine product operation matrix respectively [11].

Stability is determined by examining the largest real part of the eigenvalues. If at least one such real part is positive in sign, the system is unstable.

3 Results and Discussion

A viscoelastic shaft ,with disc placed at a specified location 'a' from one end of the spinning massless shaft, is considered with two different support conditions- simple support and both end fixed support. The end of the shaft at the precession axis is always fixed. If the other end is axially free, the shaft is termed as an axially free shaft. When both the ends are axially fixed, the shaft is referred to as an axially fixed shaft.

The shaft is 0.2m in length and 0.02m in diameter. The diameter and mass of the disc are 0.15m and 0.25kg respectively.

Four element type model of Fig.4 was found to fit reasonably to the properties [12].

$\eta_2' = 282 N.Sec/m^2, \eta_3' = 225 N.Sec/m^2 \; E_3' = 845000 N/m^2, E_1' = 2460000 N/m^2$

Values of different coefficients as mentioned above are derived from the plots of storage modulus and loss coefficient for a four element viscoelastic model [12].

Three case studies are considered where the disc is located at a = 0.03m, 0.05m and 0.08m from the precession axis. Fig.(4a) displays variation in axial centrifugal force acting on the disc due to its placement at different locations from precession axis.

<u>Axially free rotor</u>

The end at the precession axis is axially restrained while the other one is kept axially free. Stable and unstable zones of operation for the three cases are plotted in Fig.(5) and Fig.(6) for varying spin and precession speeds for a simple supported rotor and both end fixed rotor when only one end of the shaft located at the precession axis is axially restrained.

It is seen that in each case, the generic stability region is approximately of the form of a half trapezoid. Stability borderline increases with increase in precession speed due to consequent increase in axial centrifugal tension till it becomes unstable over a specific precession speed where the precession softening offsets the centrifugal stiffening.

It is seen that rotors which are subjected to larger axial tensile forces are more stable. Consequently, rotors where the disc is placed at a greater axial distance from the precession axis have comparatively larger stability zones. In lower precession speed ranges, effect of axial centrifugal force becomes less and hence there is not much contribution of centrifugal force on stability borderlines irrespective of the location of the disc.

Axially fixed rotor

Fig.(4b) displays variation in axial centrifugal force acting on the disc due to its placement at different locations from precession axis when both ends of the shaft are axially fixed. Stable and unstable zones of operation for the three case studies are plotted in Fig.(7) and Fig.(8) for varying spin and precession speeds. When the shaft is axially fixed at both ends, axial centrifugal compressive force comes into existence at one segment of the shaft in addition to the centrifugal tensile force existing at the other segment. This centrifugal compression force partially nullifies the stability leverage when the axial tensile force only used to exist in the previous case.

It is clearly demonstrated that stability zones for axially free configuration is much larger compared to that of axially fixed configuration. This shows that in order to utilise the advantage of larger stability zone one has to keep the shaft axially free at one of its two ends.

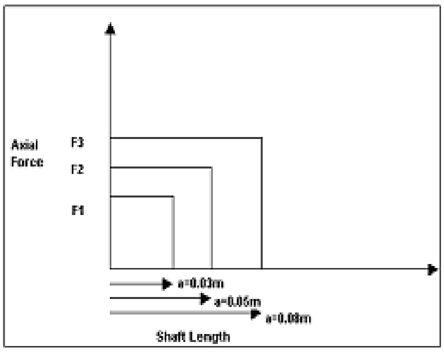

Figure.4a: Centrifugal force variation for axially free rotor

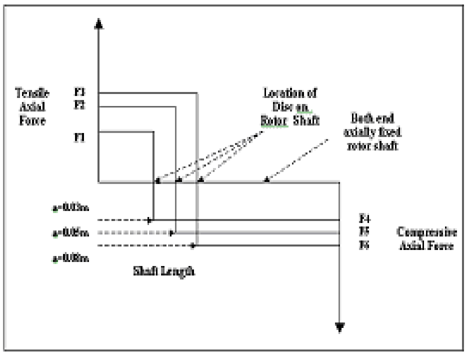

Figure.4b: Centrifugal force variation for axially fixed both end rotor

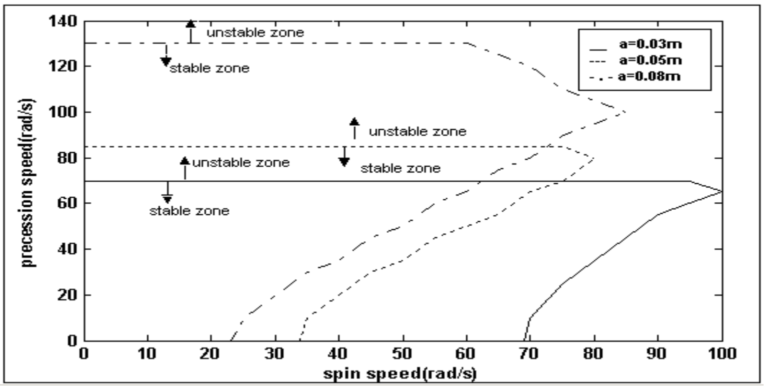

Figure.5: Plots of stable and unstable zones for axially free simple supported rotor with disc attached at distance ‘a’ =0.03m, 0.05m, 0.08m from one end

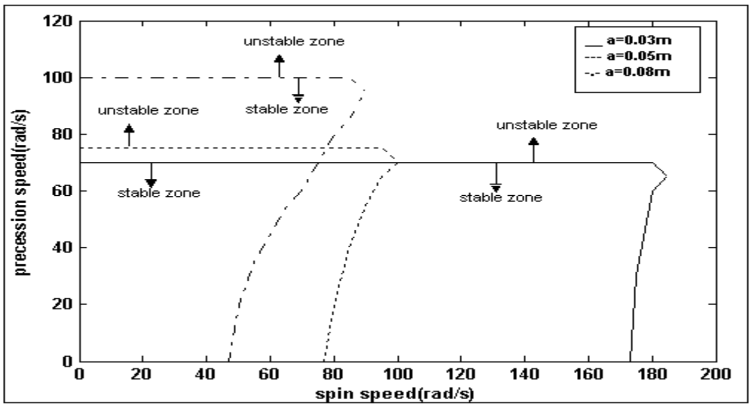

Figure.6. Plots of stable and unstable zones for axially free fixed end rotor with disc attached at distance ‘a’ =0.03m, 0.05m, 0.08m from one end

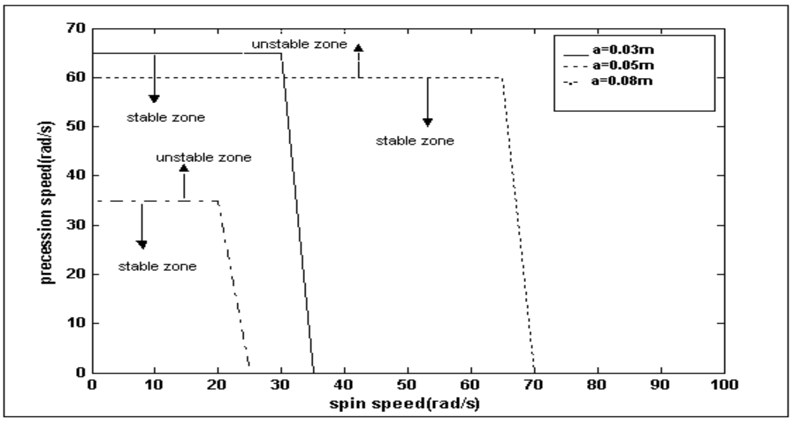

Figure.7: Plots of stable and unstable zones for axially fixed simple supported rotor with disc attached at distance ‘a’ =0.03m, 0.05m, 0.08m from one end

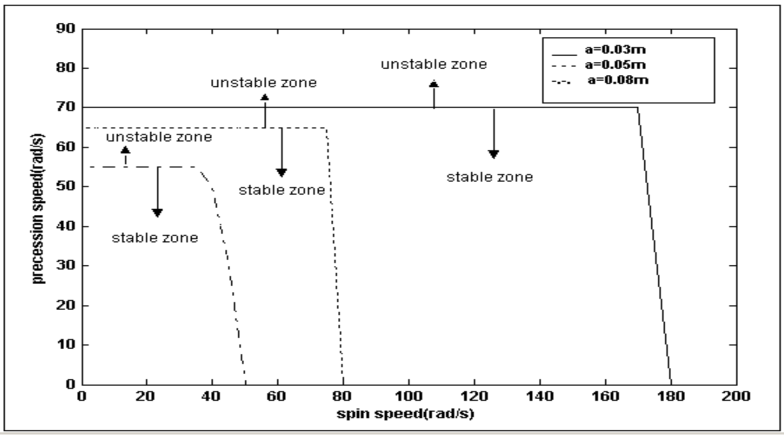

Figure.8. Plots of stable and unstable zones for axially fixed both end fixed rotor with disc attached at distance ‘a’ =0.03m, 0.05m, 0.08m from one end

4 Conclusions

The present work computes the stability borderline of a spinning and precessing viscoelastic rotor under the application of axial centrifugal force. A four-element linear viscoelastic model represents the frequency dependent material properties of the shaft. Governing equations of motion for a rigid disc on a viscoelastic shaft are derived in the rotating frame. A systematic scheme has been suggested for eigenvalue analysis of resulting 3rd order parametric equations for such systems. The effect of position of the disc and axial constraint of the shaft on stability of the rotor has been demonstrated.

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