General Method of Optimal Kinematic Synthesis of Planar Lever Mechanisms based on its Structural Properties by Example of the Eight-link Mechanism

N.N. Krokhmal, O.N. Krokhmal

Abstract

The general method of optimization kinematic synthesis by example of the 8-link planar lever mechanism is considered. The mechanism is intended for reproduction of the set movement of a target link and trajectory of the specified point. Synthesis is carried out on the basis of structural properties of the mechanism.

Keywords: the planar lever mechanism, mechanism's structure, optimization kinematic synthesis.

1 Introduction

The problem of the synthesis of high-class lever mechanisms is of great interest because such mechanisms are used widely in robotics and processing equipment [1]. However, at present there is no a general method of the kinematic synthesis of such mechanisms. Only special methods are developed.

The researches in the field of structural analysis and synthesis of lever mechanisms which we have carried out give a possibility to develop a general method of the kinematic synthesis of the mechanisms.

The problem of the kinematic synthesis of lever mechanisms, generally, is to define geometry of the mechanisms with the purpose to provide the required motion of a certain link or the required path of motion of a certain point lies at any link of the mechanism.

In the scheme shown in Fig.(1) the joints of the mechanism are marked with number, and the levers are marked with two numbers which are the numbers of the hinges forming this lever.

Position of the input link (3-4-10) is specified by generalized coordinate – an angle ($\varphi+\varphi_0$). It is necessary to define lengths of all the levers and angels of initial positions of input (φ_0) and output (θ_0) links. The point 0 must move along the required trajectory described by the function S=S(φ). The output link (7-8-9) must rotate corresponding to the required law of motion $\theta=\theta(\varphi)$, when $\varphi_0\le\varphi\le\varphi_{max}$.

In [2] it is shown that any lever mechanism can be formed as a kinematic system of dyads and some number of input (output) levers. This number is corresponding to degree of mobility of a mechanism.

As it known, a dyad is the simplest kinematic chain which consists of two levers, which form a hinge. In Assur groups dyads have feedforwards and feedbacks between each other.

N.N. Krokhmal
Kurgan state university , Russia, Kurgan, kroknn@yandex.ru

O.N. Krokhmal
Kurgan state university , Russia, Kurgan, olkron@mail.ru

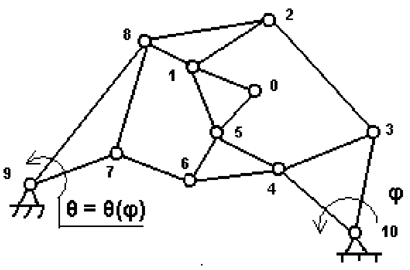

Figure 1: The structure scheme of the lever mechanism.

In this sample the mechanism is formed by the chain of dyads shown in Fig.(2), and the lever 89.

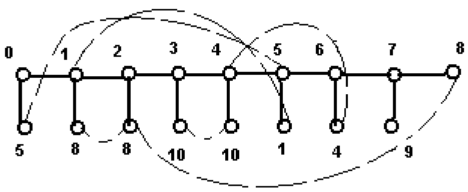

Figure 2: The scheme of connections of dyads between themselves in the mechanism

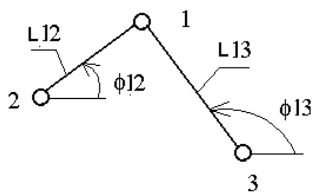

Figure 3: The scheme of a dyad.

Any dyad is an elementary transforming mechanical system which has two inputs (external kinematic pairs) and one output (internal kinematic pair). Signals (speeds, displacements) come on inputs with starting values and then they are taken down with transformed values (Fig. (3)). Thus, it is possible to tell that a dyad is the elementary element of mechanical system of automatic control which is Assur group. Any dyad has a transfer function of certain kind (Eq.(1)).

$$\begin{bmatrix} x1' \\ y1' \end{bmatrix} = [J2] \times \begin{bmatrix} x2' \\ y2' \end{bmatrix} + [J3] \times \begin{bmatrix} x3' \\ y3' \end{bmatrix}, \qquad (1)$$

where $\begin{bmatrix} xi' \\ yi' \end{bmatrix}$ - projections of velocity (slight displacements) of a point i of the dyad in the specified Cartesian coordinate system, $[J2] = \begin{bmatrix} i1 & i2 \\ i3 & i4 \end{bmatrix}$, $[J3] = \begin{bmatrix} i4 & -i2 \\ -i3 & i1 \end{bmatrix}$ - matrixes of return transfer functions of the dyad from point 2 to point 1, and from point 3 to point 1.

The components of the matrixes can be defined as follows:

$$i1 = \frac{\boldsymbol{tan}\,\phi 13}{\boldsymbol{tan}\,\phi 13 - \boldsymbol{tan}\,\phi 12}, \qquad i2 = \frac{\boldsymbol{tan}\,\phi 12 \cdot \boldsymbol{tan}\,\phi 13}{\boldsymbol{tan}\,\phi 13 - \boldsymbol{tan}\,\phi 12},$$

$$i3 = \frac{-1}{\boldsymbol{tan}\,\phi 13 - \boldsymbol{tan}\,\phi 12}, \qquad i4 = \frac{-\boldsymbol{tan}\,\phi 12}{\boldsymbol{tan}\,\phi 13 - \boldsymbol{tan}\,\phi 12}.$$

Tangents in these equations can be expressed through coordinates of points where joints of a dyad are placed.

The general transfer function of the mechanism can be expressed through the transfer functions of all dyads which the mechanism consists of (Eq.(2)).

$$\left.\begin{array}{ll}
\begin{pmatrix} x7' \\ y7' \end{pmatrix} = J17 \cdot \begin{pmatrix} x8' \\ y8' \end{pmatrix} & \begin{pmatrix} x3' \\ y3' \end{pmatrix} = J5 \cdot \begin{pmatrix} x4' \\ y4' \end{pmatrix} \\
\begin{pmatrix} x6' \\ y6' \end{pmatrix} = J8 \cdot \begin{pmatrix} x7' \\ y7' \end{pmatrix} + J15 \cdot \begin{pmatrix} x4' \\ y4' \end{pmatrix} & \begin{pmatrix} x2' \\ y2' \end{pmatrix} = J4 \cdot \begin{pmatrix} x3' \\ y3' \end{pmatrix} + J11 \cdot \begin{pmatrix} x8' \\ y8' \end{pmatrix} \\
\begin{pmatrix} x5' \\ y5' \end{pmatrix} = J7 \cdot \begin{pmatrix} x6' \\ y6' \end{pmatrix} + J14 \cdot \begin{pmatrix} x1' \\ y1' \end{pmatrix} & \begin{pmatrix} x1' \\ y1' \end{pmatrix} = J3 \cdot \begin{pmatrix} x2' \\ y2' \end{pmatrix} + J10 \cdot \begin{pmatrix} x8' \\ y8' \end{pmatrix} \\
\begin{pmatrix} x4' \\ y4' \end{pmatrix} = J6 \cdot \begin{pmatrix} x5' \\ y5' \end{pmatrix} & \begin{pmatrix} x0' \\ y0' \end{pmatrix} = J2 \cdot \begin{pmatrix} x1' \\ y1' \end{pmatrix} + J9 \cdot \begin{pmatrix} x5' \\ y5' \end{pmatrix}
\end{array}\right\} \quad (2)$$

So the general transfer function is a system of ordinary differential equations (ODE) relative to coordinates of the points where joints are located. As a result of the solving the ODE (the of Cauchy problem) paths of motion of the kinematic pairs can be defined. As it is known, for solving Cauchy problem it is necessary to set initial conditions for the ODE. In this case it is necessary to set initial values of coordinates of points where kinematic pairs are located.

At carrying out of kinematic synthesis of the mechanism it is possible to define comprehensible intervals of possible initial conditions for solving the ODE. Thus, the problem of kinematic synthesis of a mechanism is to select such initial conditions from the given intervals at solving ODE, which would provide required paths of motion of kinematic pairs and constant lengths of all levers of the mechanism. The choice of initial conditions of integration of a system can be carried out by a method of global optimization of the special objective function.

2 The Algorithm of an Objective Function Computing

An objective function is the major element for the solving an optimization problem. Generally, this function is implicit and its value is defining by calculations. For the considering example the algorithm is the following:

1. Specify a number of positions which to be calculated - n
2. Specify (corresponding to 1) rotation angles (generalized coordinate) of the input link (3-4-10)
3. Choose initial values of coordinates (xN_0, yN_0) of the points with numbers N from 1 to 10. Coordinates of the point 0 is known and defined by its law of motion.
4. Calculate angles which fix the initial positions of the levers form the input and output links.

$$\phi 310 = \boldsymbol{arctan(}\frac{y3_0 - y10_0}{x3_0 - x10_0}\boldsymbol{)} \qquad \phi 410 = \boldsymbol{arctan(}\frac{y4_0 - y10_0}{x4_0 - x10_0}\boldsymbol{)}$$

$$\vartheta 79 = \boldsymbol{arctan(}\frac{y7_0 - y9_0}{x7_0 - x9_0}\boldsymbol{)} \qquad \vartheta 89 = \boldsymbol{arctan(}\frac{y8_0 - y9_0}{x8_0 - x9_0}\boldsymbol{)}$$

5. Calculate the lengths of the levers for the particular position (i) on the basis of constraint equations between points N and M.

$$LNM_i = \sqrt{(xN_i - xM_i)^2 + (yN_i - yM_i)^2}\ .$$

For the levers 310, 410,34, 79, 89 and 78 the equation $LNM_i = LNM_0$ is correct because motion paths of points of this levers are always circles.

6. Calculate displacements of the points when they move from position (i) to position (i+1). In this mechanism these points are: 0, 3, 4, 7, 8. Calculate slight displacements (as projections onto the axes of coordinates) when mechanism moves from the current position to next one. Calculating positions (step 1, 2) was chosen in such a way that they correspond to small changes of the generalized coordinate. The small changes are the values which are typical for steps of numerical integration by Euler method.

Point 0 $\Delta x0 = x0(\phi_i + \Delta\phi) - x0(\phi_i)$,

$\Delta y0 = y0(\phi_i + \Delta\phi) - y0(\phi_i)$.

Point 3 $\Delta x3 = L310 \cdot (\boldsymbol{cos}(\phi 310 + \phi_i + \Delta\phi) - \boldsymbol{cos}(\phi 310 + \phi_i))$,

$\Delta y3 = L310 \cdot (\boldsymbol{sin}(\phi 310 + \phi_i + \Delta\phi) - \boldsymbol{sin}(\phi 310 + \phi_i))$.

Point 4 $\Delta x4 = L410 \cdot (\boldsymbol{cos}(\phi 410 + \phi_i + \Delta\phi) - \boldsymbol{cos}(\phi 410 + \phi_i))$,

$\Delta y4 = L410 \cdot (\boldsymbol{sin}(\phi 410 + \phi_i + \Delta\phi) - \boldsymbol{sin}(\phi 410 + \phi_i))$.

Point 7 $\Delta x7 = L79 \cdot (\boldsymbol{cos}(\vartheta 79 + \vartheta(\phi_i + \Delta\phi)) - \boldsymbol{cos}(\vartheta 79 + \vartheta(\phi_i)))$,

$\Delta y7 = L79 \cdot (\boldsymbol{sin}(\vartheta 79 + \vartheta(\phi_i + \Delta\phi)) - \boldsymbol{sin}(\vartheta 79 + \vartheta(\phi_i)))$.

Point 8 $\Delta x8 = L89 \cdot (\boldsymbol{cos}(\vartheta 89 + \vartheta(\phi_i + \Delta\phi)) - \boldsymbol{cos}(\vartheta 89 + \vartheta(\phi_i)))$,

$\Delta y8 = L89 \cdot (\boldsymbol{sin}(\vartheta 89 + \vartheta(\phi_i + \Delta\phi)) - \boldsymbol{sin}(\vartheta 89 + \vartheta(\phi_i)))$.

7. Calculate displacements of other points (1, 2, 5, 6) when they move from position (i) to position (i+1) by solving the system of linear equations (2). In this case the equations 1 and 5 mast be eliminated from the system (2) because they have no unknown displacements.

8. Calculate coordinates of the points for (i+1) position of the mechanism using Euler formulas: $xN_{i+1} = xN_i + \Delta xN_i \quad yN_{i+1} = yN_i + \Delta yN_i$

9. Repeat steps 5-8 for all positions of the mechanism and whole interval of the generalized coordinate (φ).

10. Calculate average lengths of levers NM using the results of calculations for all mechanism positions.

$$midLNM = \frac{\sum_{i=0}^{n} LNM_i}{n+1}$$

11. Calculate deviations of levers lengths from middle value for each mechanism position $\Delta LNM_i = midLNM - LNM_i$

12. Calculate a value of the objective function as the sum of standard deviations of all levers lengths in all mechanism positions.

$$Q = \sum_{1}^{11} \sqrt{\frac{\sum_{i=0}^{n} (\Delta LNM_i)^2}{n+1}} ,$$

here 11 – the number of levers with non-stationary lengths, n – the number mechanism positions need to be calculated.

After we get a rule of calculating of an objective function, the problem of a mechanism synthesis comes to finding arguments of the objective function which are initial values of points coordinates when the function has its minimum. It is desirable that this minimum would be global and of admissible value. This value corresponds to acceptable deviations of lever lengths and admissible accuracy of needed laws of motion of points and levers.

In practice, to solve this problem is effective to use genetic algorithms. These methods base on biological evolution theory and the breeding experience [3]. A search strategy of genetic algorithms base on selection hypothesis: the higher adjustment of an individual the higher probability that its progeny will have stronger characters of adjustment [4].

In [5] the genetic algorithm and its software realization are shown. This algorithm has genetic properties of statistic selection of searching point’s population. To exclude “poor” progeny the special procedure is used. The procedure calculates local extremums by using operations of deformable polygon. When generations alternate the algorithm replaces 10% of “poor” individuals that is recommended in many papers. This algorithm and its realization as MathCad program was used for the synthesis of the sample mechanism.

3 The Example of the Mechanism Synthesis

The data are taken from [6]. For this mechanism it is required to make up the follow laws of motion:

Table 1: Required laws of motion

An angular position of input link φ, deg	0	45	55	60	65	69	73
An angular position of output link θ. deg	0	19	33	44	52	58	62
X-coordinate of the point 0	2,18	2,14	1,98	1,80	1,60	1,42	1,27
Y-coordinate of the point 0	0,84	1,07	1,21	1,37	1,52	1,64	1,72

Using the input data the linear interpolation is carried out. As a result the following functions are obtained: $\theta = \theta(\varphi)$, $x0 = x0(\varphi)$ and $y0 = y0(\varphi)$.

In optimization searching 74 computing positions of the mechanism was used ($n=0\ldots74$, step of generalized coordinate $\Delta\varphi = 1^0$).

In the program of global optimization the following input data was used: a number of arguments of the objective function is 20, a number of individuals in a population is 9, a precision of calculation of the objective function is 10^{-2}, initial searching intervals for all arguments of the objective function are -3,5…3,5.

A 2.66 GHz computer found the minimum (Q=0.124929) of the objective function in 20 minutes. Also was found the values of arguments of the objective function and average lengths of all the levers of the mechanism. The found arguments were improved by MathCad standard procedure of local minimum searching. This program improved the minimum (Q = 0.032641), the values of arguments (Table 2) and average lengths of the levers (Table 3) in 8 minutes.

Table 2: Initial coordinates of the mechanism's points

№	0	1	2	3	4	5	6	7	8	9	10
X	2,170	0,710	0,590	-0,250	-0,360	-0,690	-0,870	-0,260	0,170	-0,160	-0,410
Y	0,843	-0,255	-0,218	-0,986	-1,280	-0,928	-1,104	0,937	0,252	0,679	-0,878

Table 3: Lengths of the levers of the mechanism

Lever	L01	L12	L23	L34	L45	L56	L67	L78	L89
Length	1,832	0,122	1,149	0,310	0,486	0,251	2,132	0,812	0,546
Lever	L05	L18	L28	L310	L410	L51	L64	L79	-
Length	3,378	0,736	0,628	0,188	0,405	1,562	0,544	0,270	-

Deviations of lengths of the levers in computed positions have an order about 10^{-8}.

To verify the calculations the paths of motion of joints was found when lengths of the levers are known. For this purpose the standard MathCad program for solving systems of non-linear equations was used. The kind of equations is $LNM_i^2 = (xN_i - xM_i)^2 + (yN_i - yM_i)^2$. The diagram of deviations of point 0 coordinates from its required law of motion is shown in Fig. (4).

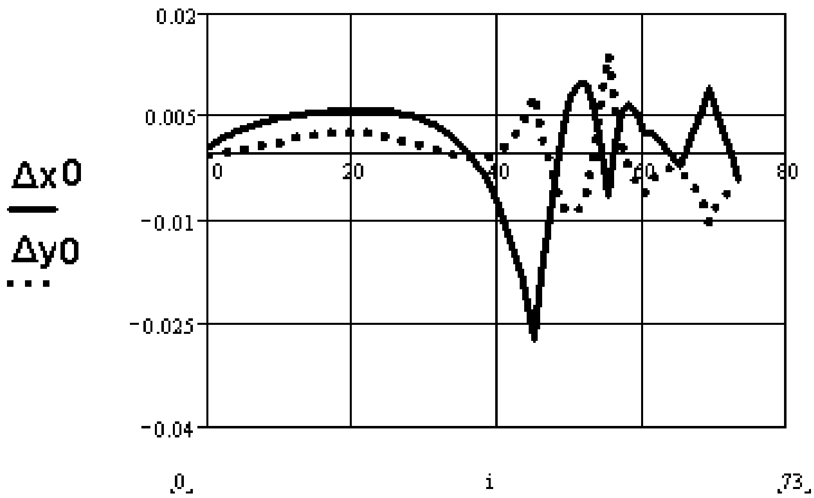

Figure 4: The diagram of deviations of point 0 coordinates from its required law of motion

The diagram of deviation of rotation angle of output link from its required law of motion is shown in Fig. (5).

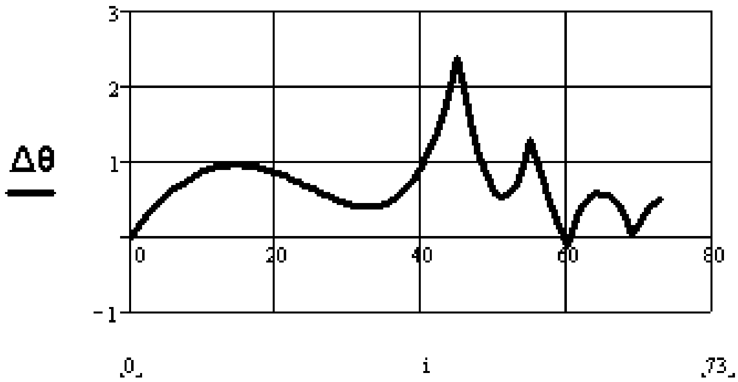

Figure 5: The diagram of deviations (degree) of rotation angle of output link from its required law of motion

The kinematic scheme of the mechanism which was generated is shown in Fig.(6). The scheme shows also the paths of motion of the mechanism points where kinematic pairs are placed.

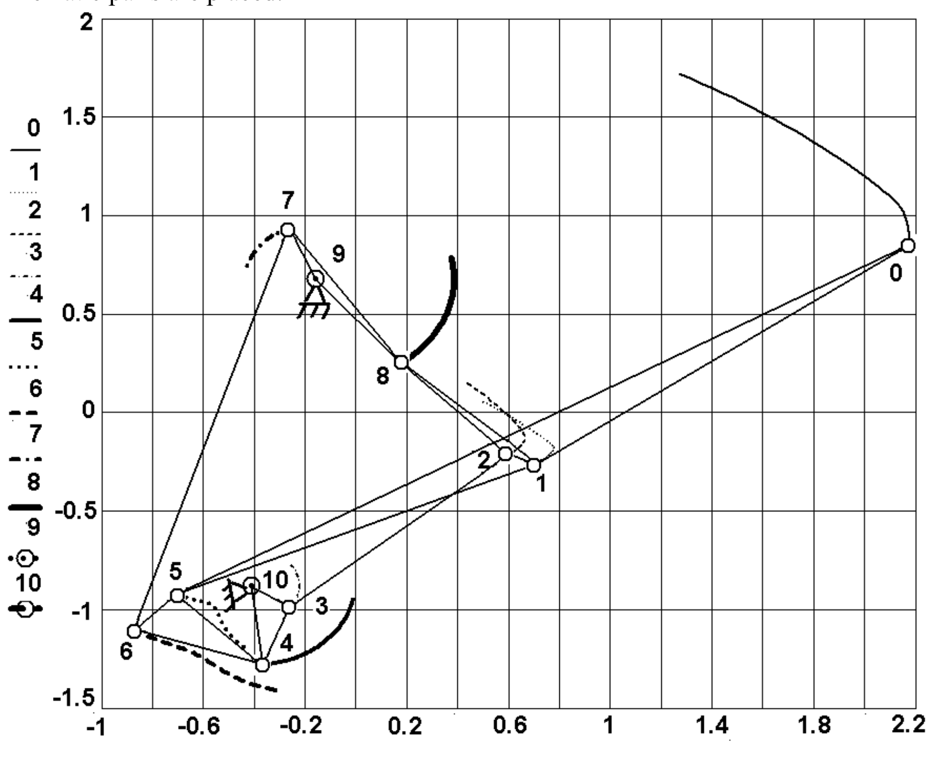

Figure 6: The kinematic scheme of the generated mechanism (starting position)

4 Conclusions

The suggested method can be used in optimization synthesis of any lever mechanism, because it based on general structure properties of such mechanisms – dyad structure. The method of optimization synthesis presupposes the searching of global minimum of objective function. Modern algorithms of global optimization of functions permit to do it successfully. A Further developing of this method and adaptation for it of the global optimization method will allow building up a holistic system of kinematic synthesis of lever mechanisms of any class with required precision.

References

[1] Merlet J. P and Dahan M., 12th World congress on the TMM. Basancon, France, June 17-20, 2007.

[2] Krokhmal N. Structural synthesis of kinematic chains of lever mechanisms, 13th National Conference on Mechanisms and Machines (NaCoMM07), IISc, Bangalore, India, December 12-13, 2007 – pp.149-155.

[3] Goldberg D.E., Genetic Algorithms in Search Optimizations and Machine Learning.-Addison.Wesly, 1989.

[4] G.K.Voronovskij, K.V. Mahotilo, S.N. Petrashev, S.A. Sergeev. Geneticheskie algoritmy, iskusstvennye nejronnye seti i problemy virtual'noj real'nosti, Harkov, Osnova, 1997 (in russian).

[5] Sabanin V.R., Smirnov N.I., Repin A.I. Modificirovannyj geneticheskij algoritm dlja zadach optimizacii v upravlenii, Exponenta Pro, 2004. V 3-4. pp. 78-85 (in russian).

[6] S. Hamid, A.H. Soni Synthesis of an Eight-Link Mechanism for Varieties of Motion Programs, Transactions of the ASME, journal of engineering for industry, 1973. No 3.C. 76-84.