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Singularity Analysis of Closed-loop Mechanisms and Parallel Manipulators

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Abstract

Singularity analysis is the study of gain or loss in degrees of freedom of a mechanism at a particular configuration. Singularity analysis is related to the degeneracy of Jacobian matrices relating configuration variables with task space variables. In this paper, we present an improved Jacobian formulation which can be used to identify not only the gain or loss of degrees of freedom, but, in addition, can be used to determine if the gain results in redundant degree of freedom. The approach and its advantages are illustrated with several examples.

Keywords: Degree of freedom (DOF), Manipulator, Kinematics, Singularity, Actuated, Passive joints, Redundant

1 Introduction

At a singular configuration, a closed loop mechanism or a parallel manipulator can loose or gain one or more degrees of freedom (DOF). At such a configuration, Jacobian matrices associated with the first-order (velocity) kinematics of a manipulator becomes degenerate. There are two main approaches adopted by researchers to analyse the singularities in closed-loop mechanisms and parallel manipulators. In the well-known approach by Gosselin and Angeles [1], the first-order kinematics are written in terms of derivatives of chosen task space variables and input joint variables and using Jacobian matrices associated with these chosen variables. The loss of rank of the matrices leads to loss, gain or architectural singularities. In an alternate approach [2], first the constraint equations are obtained and then the relationship between the actuated and passive joint rates are obtained by differentiating the constraint equations. The loss of rank of the Jacobian matrix associated with the passive joint variables are related to *gain* of one or more DOF. The two methods have their own advantages and

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disadvantages – in the first approach, the *choice* of the output link must be made before the analysis. In the second approach, the gain of DOF is obtained for *locked* actuated joints. In addition, for both the approaches, the nature of the gained DOF is not very clear. For example, consider a case of the well-known *translational* 3-UPU manipulator [3]. It is not clear if at a *gain singularity* the gain is in linear velocity or the gain is an angular DOF. If the gain is in linear velocity the manipulator becomes instantaneously redundant and the behaviour could be very different if the gain is in angular velocity. In this paper, we attempt to answer these issues by developing a modified Jacobian based approach. We show with help of several illustrations that the modified Jacobian approach enables us to determine if the gain of one or more DOF results in the manipulator becoming redundant or not.

The paper is organized as follows: In the next section, we review the existing approaches for *gain* singularity analysis in closed-loop mechanisms and parallel manipulators. We next present the modified Jacobian based approach. In section 3, we present several examples, both planar and spatial, which illustrate the advantages of the modified Jacobian approach. The conclusions are present in section 4.

2 Existing and the Modified Jacobian Approach

In the approach by Gosselin and Angeles [1], the first step is to obtain an equation of the form

$$[A]\dot{x} + [B]\dot{l} = 0 \tag{1}$$

where x is a vector of chosen task space variables and l are the input joint variables. The loss of rank of $[A]$ and $[B]$ gives rise to two different kinds of singularities, namely those associated with gain and loss of one or more degree of freedom (DOF), and when both $[A]$ and $[B]$ looses rank, the mechanism is said to be in an *architectural* singularity. In the constraint equation based approach, the loop-closure constraint equations are written in the form

$$\eta_i(l, \theta) = 0 \tag{2}$$

where the active and passive joint variables are denoted by l and θ, respectively. From the above, differentiating with respect to time, we get

$$[K]\dot{l} + [K^*]\dot{\theta} = 0 \tag{3}$$

At a *non-singular* configuration, $\det[K^*] \neq 0$, the the angular velocity of the (chosen) output link or platform, $\{Tool\}$ with respect to a fixed reference $\{0\}$ can be obtained as

$${}^0\omega_{Tool} = [J_\omega]_{eq}\dot{l} \tag{4}$$

where $[J_\omega]_{eq}$ is obtained after eliminating the passive joint rates, using Eq.(3). In the reference [4], the eigenvalues of $[J_{\omega eq}]^T[J_{\omega eq}]$ are used for *partitioning* the degrees of freedom of the chosen output link $\{Tool\}$. At a singular configuration, $\det[K^*] = 0$ and the *gain* in DOF is characterised by the number of non-zero eigenvalues of $[J^*_\omega]^T[J^*_\omega]$ where $[J^*_\omega]$ is the Jacobian associated only with the passive variables with the actuator locked $(\dot{l} = 0)$.

2.1 Modified Jacobian formulation

As mentioned earlier, in the first approach, the choice of the task space variables determines what can be readily inferred from the degeneracy of $[A]$ and $[B]$. In the constraint equation based, $\dot{l}$ is set to zero for determining gained DOF. In both the approaches, the nature of the gained DOF is not very clear and we cannot determine if the gain results in a redundant system. To answer this and similar questions, we aim to write the first-order kinematics in the form

$$[J]^0\omega_{Tool} + [J_\theta]\dot{\theta} = 0 \tag{5}$$

where ${}^0\omega_{Tool}$ is the angular velocity of the output link and θ denotes the vector of *passive* variables. From Eq. (5) if $[J]$ becomes rank deficient in the 3-UPU case, the gain is in *linear* velocity which makes the 3-UPU translational manipulator redundant. If the rank of $[J]$ does not change, then the gained DOF is in angular velocity. As in the constraint equation based approach, the results are based on the angular velocity and hence are independent of the point on the moving output link or platform.

To form Eq. (5) we first derive the angular and linear velocity of $\{Tool\}$ with respect to a fixed coordinate system $\{0\}$ as

$$\begin{aligned} {}^0\omega_{Tool} &= [J_\omega]\dot{l} + [J_\omega^*]\dot{\theta} \\ {}^0V_{Tool} &= [J_v]\dot{l} + [J_v^*]\dot{\theta} \end{aligned} \tag{6}$$

Next, by differentiating *inverse* kinematic equations we get

$$[J_a]\begin{bmatrix} {}^0V_{Tool} \\ {}^0\omega_{Tool} \end{bmatrix} = [J_l]\dot{l} \tag{7}$$

Using Eq. (6) and Eq. (7), one can eliminate actuated joint rate as

$$\dot{l} = ([J_l] - [J_{a_1}][J_v] - [J_{a_2}][J_\omega])^{-1}([J_{a_1}][J_v^*] + [J_{a_2}][J_\omega^*])\dot{\theta} \tag{8}$$

and the above equation can be rewritten as

$${}^0\omega_{Tool} = [C]\dot{\theta}$$

where $[C]$ is given by

$$[C] = [J_\omega]([J_l] - [J_{a_1}][J_v] - [J_{a_2}][J_\omega])^{-1}([J_{a_1}][J_v^*] + [J_{a_2}][J_\omega^*]) + [J_\omega^*]$$

The matrix $[C]$ on the right-hand side need not be square but we can rewrite above equation as

$$[C]^T({}^0\omega_{Tool}) = [C]^T[C]\dot{\theta} \tag{9}$$

and identifying $[C]^T$ with $[J_\omega]$ and $-[C]^T[C]$ with $[J_\theta]$, we get the desired first-order kinematics described in equation (5). In the next section, we use the above form of the first-order kinematics to analyse gain singularity.

3 Results and Discussion

We present numerical examples for two and three degrees of freedom planar and three and four degrees of freedom spatial manipulators.

3.1 Two DOF planar manipulator

We start with a planar 2 DOF manipulator shown in Fig. 1. It has two actuated joints namely A_1, A_2 and six passive joints. The output link is a triangular platform.

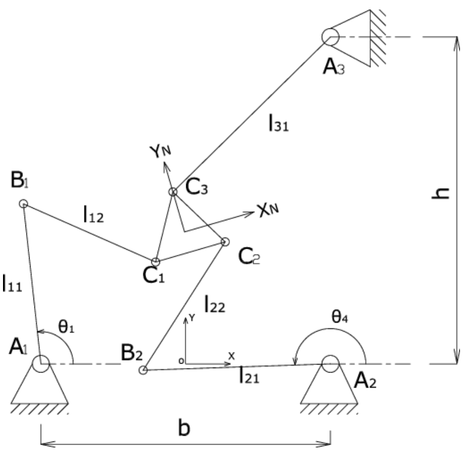


Figure 1: A 2 DOF planar parallel manipulator

For the planar manipulator, the geometric parameters are chosen as $b = 1$, $h = 1.5$, $a = 0.3$, $l_{11} = l_{21} = 0.5$, $l_{12} = l_{22} = 0.5$, $l_{31} = 1.2$. At non-singular configuration, the partition in DOF can be shown to be 2 linear velocity and zero angular velocity from the constraint equation based approach. For the actuated variable values [1] $\theta_1 = \pi/2$, $\theta_4 = 1.3143$ and for the passive variable values $\theta_2 = -2.0628$, $\theta_3 = 2.0927$, $\theta_5 = 1.3318$, $\theta_6 = 1.0489$, $\theta_7 = -1.7779$ and $\theta_8 = 4.0703$, the manipulator gains a degree of freedom. Since in the non-singular case, the DOF are translational, the gain DOF can be concluded to be in angular velocity. However, if we use the modified Jacobian approach and rewrite the first-order kinematics as

$$[J]\omega_z + [J_\theta]\dot{\phi} = 0 \tag{10}$$

we see that $[J]$ (in this case scalar) is very close to zero at the gain singular configuration. Hence, at this singular configuration, the gain *is not in angular velocity* but in linear velocity, i.e., the planar mechanism is *instantaneously redundant*.

3.2 Three DOF planar manipulator

We next consider the three DOF planar manipulator shown in Fig. 2. It has three actuated joints A_1, A_2 and A_3 and six passive joints. The output link is a triangular platform. At a non-singular configuration, the manipulator *must* have two linear and one angular velocity component. For the geometric parameters, $a = 0.6$, $h = 0.3$, $l_{11} = l_{21} = 0.5$, $l_{12} = l_{22} = 0.5$, $l_{31} = l_{32} = 0.5$, and actuated variables $\theta_1 = \pi/2$,

[1] All angles are in radians.

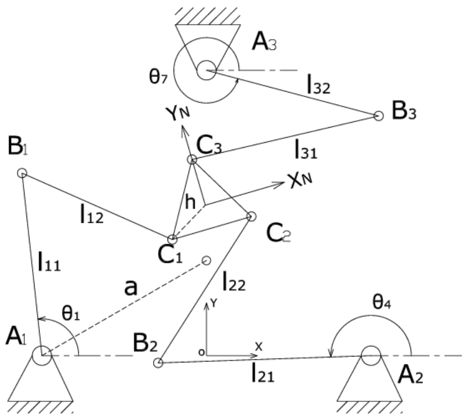


Figure 2: A 3 DOF planar parallel manipulator

$\theta_4 = 2.5$, $\theta_7 = -0.7622$ and passive variables $\theta_2 = -0.5374$, $\theta_3 = -1.8048$, $\theta_5 = 0.0826$, $\theta_6 = -2.0359$, $\theta_8 = 2.8834$, and $\theta_9 = -4.2107$, the 3 DOF planar manipulator gains a DOF. The existing approaches for analysis of gain singularity cannot be used to identify nature of the gained velocity and identify the manipulator is *redundant in terms of linear or angular velocity*.

Using the modified Jacobian approach, we can write two Jacobian relationships for the angular velocity and linear velocity in terms of the passive variables. These are of the form

$$[J]\omega_z + [J_\theta]\dot{\phi} = 0, \quad [J_1]V + [J^*_\theta]\dot{\phi} = 0 \tag{11}$$

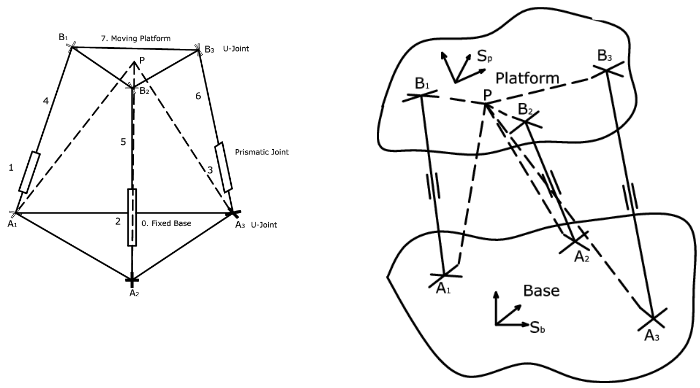
Computing $[J]$ and $[J_1]$ at the gain singular configuration, we find that $[J_1]$ is very close to zero whereas $[J]$ is not. Hence at this gain singular configuration, the gain is *not in angular velocity* but in linear velocity, and the planar manipulator is *instantaneously redundant in linear velocity*.

3.3 The 3-UPU wrist manipulator

The 3-UPU manipulator [5] shown in Fig. 3 is a well-known three DOF parallel manipulator. The point 'P' is the common intersection of the revolute pair axes fixed in the platform. Mounting condition requires that platform point 'P' coincides with base point located by the intersection of revolute pair axes fixed in the base. For base platform length $b = 1$ and top platform length $a = 1$, at a non-singular configuration the *DOF partition is 3 angular velocity (ω) and 0 linear velocity (V)*. For $l_1 = 0.1$, $l_2 = 0.5$, $l_3 = 1.9854$ and passive variable values of $\theta_1 = 0.3361$, $\theta_2 = 1.5208$, $\theta_3 = 1.5208$, $\theta_4 = -0.0864$, $\theta_5 = 4.9651$, $\theta_6 = -1.3181$, $\theta_7 = 0.4679$, $\theta_8 = -0.1210$ and $\theta_9 = -0.1210$, the 3-UPU gains a degree of freedom. To figure out if the gained DOF is in angular or linear velocity, we obtain the modified Jacobian equation

$$[J_1]V + [J^*_\theta]\dot{\theta} = 0 \tag{12}$$

For the gain singular configuration, the rank of $[J_1]$ is found to be 2. This implies that the gained DOF is angular and hence the 3-UPU is redundant in terms of angular degrees of freedom at the given gain singular configuration. It may be mentioned again that using the approaches of Gosselin and Angeles or the constraint equation, it is not possible to infer this result.



(a) 3 UPU wrist schematic (b) 3 UPU wrist sketch

Figure 3: The 3-UPU wrist parallel manipulator

3.4 The 3-UPU translational manipulator

Fig. 4 [3] shows a 3-UPU parallel manipulator. It consists of a fixed base and movable platform interconnected by three identical UPU kinematic chains, with prismatic joint as the actuated joints. The sufficient condition for the manipulator to perform translational motion is that first revolute axis is parallel to last revolute axis similarly second and third revolute joint axis are parallel to one another. It can also be shown using the constraint equation based approach, that the 3 DOF's at a non-singular configuration can be partitioned as 3 linear velocity (V) and 0 angular velocity (ω). For $b = 1/2$ and $a = 1$ and for $l_1 = 2$, $l_2 = 1.5$, $l_3 = 1.2868$ and with the passive joint angles as $\theta_1 = 0.5656$, $\theta_2 = -1.6566$, $\theta_3 = 1.6566$, $\theta_4 = 2.5756$, $\theta_5 = 0.9234$, $\theta_6 = -1.1028$, $\theta_7 = 1.1028$, $\theta_8 = 2.2178$, $\theta_9 = 1.1250$, $\theta_{10} = -1.9742$, $\theta_{11} = 1.9742$, and $\theta_{12} = 2.0162$, the 3-UPU manipulator is in a gain singular configuration. Again using the modified Jacobian relation, we obtain the rank of $[J]$ as 2. This implies that the gained DOF is translational and the manipulator is *redundant* at the gain singularity configuration.

3.5 The 4-UPU parallel manipulator

The parallel manipulator shown in Fig. 5 [6] has four identical limbs each consisting of four revolute joints and one prismatic joint. The axes of first and fifth joint are

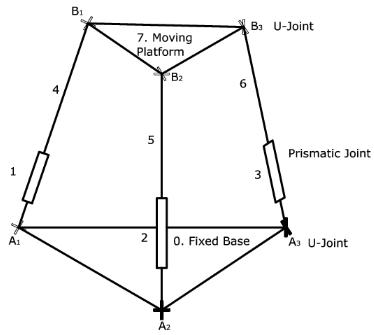


Figure 4: The 3-UPU translational manipulator

parallel to Z axis. The axes of second and the fourth joint are parallel to each other also and perpendicular to Z-axis. The axes of the fourth revolute joints in all limbs are coplanar. The base platform is rectangular while movable platform is square.

At a typical non-singular configuration the DOF is partitioned as three linear velocity (V) and one angular velocity (ω). For $R_1 = 0.49$, $R_2 = 0.39$, $d = 0.19$, and $l_1 = 0.8$, $l_2 = 1.2$, $l_3 = 2$, $l_4 = 1.6608$, $\theta_1 = 1.0119$, $\theta_2 = -0.9626$, $\theta_3 = 0.9626$, $\theta_4 = -3.6861$, $\theta_5 = 1.4806$, $\theta_6 = 1.1800$, $\theta_7 = -1.1800$, $\theta_8 = -0.4505$, $\theta_9 = 0.4902$, $\theta_{10} = 1.3402$, $\theta_{11} = -1.3402$, $\theta_{12} = -1.5864$, $\theta_{13} = -0.7643$, $\theta_{14} = 1.2920$, $\theta_{15} = -1.2920$, and $\theta_{16} = -1.3472$, the manipulator is in a singular configuration. To analyse the gained DOF, we obtain the modified Jacobian $[J]$. For this configuration rank of $[J]$ is two, indicating that there is a redundant linear velocity gain, i.e, *DOF partitioning still remains as 3 linear velocity (ω) and one angular velocity (V)*. By the constraint equation based approach, number of zero eigenvalues of $[K^*]^T[K^*]$ is two, which implies that there is two gain in DOF's. The computed eigenvalues of $[J^*_\omega]^T[J^*_\omega]$ are $[545.1, 185.75, 32.01]$ and this suggest that the gain takes place in angular velocity which is not true. Hence this example proves that the constraint equation based approach can't identify the redundant configuration.

3.6 Four DOF parallel manipulator for larger workspace

The manipulator shown in Fig. 6 was proposed by the authors [7] for obtaining large workspace, magnification ratio of Z axis and compact frame work. It is made up of moving platform, fixed base (both square) and four limbs. Limb II and IV use rhombus

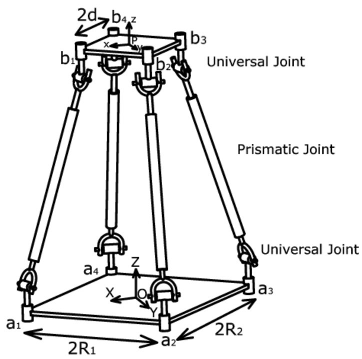


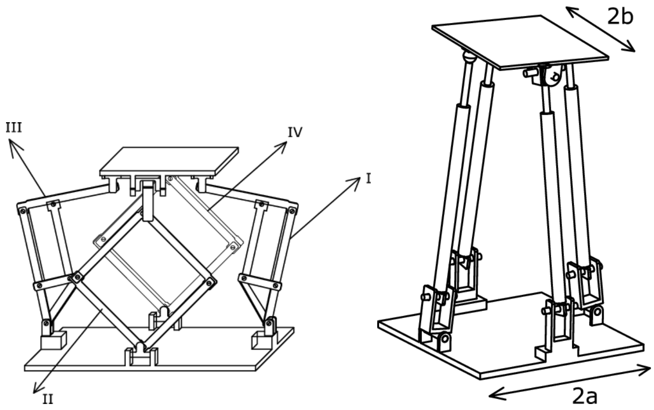
Figure 5: The 4-UPU parallel manipulator

mechanism, while limb I and III employ *scaling ruler*, which is a modified version of rhombus mechanism. By the concept of equalised representation, limb I and III is considered as RPU chain and limb II and IV as UPS chain.

For non-singular configuration the DOF is partitioned as 2 angular velocity (ω) and 2 linear velocity (V). For the base platform dimension of $2a = 0.3$ and movable platform dimension of $2b = 0.2$, and for $l_1 = 0.3$, $l_2 = 0.4$, $l_3 = 0.5$, $l_4 = 0.4026$, $\theta_1 = 0.3994$, $\theta_2 = -1.2272$, $\theta_3 = -0.0734$, $\theta_4 = -2.5108$, $\theta_5 = 0.71146$, $\theta_6 = 0.9851$, $\theta_7 = 1.3287$, $\theta_8 = 0.0734$, $\theta_9 = 11.8625$ and $\theta_{10} = 0.6222$, the manipulator is in a gain singular configuration. To analyse the nature of the gained motion, we obtain the modified Jacobian $[J]$ and in this case, the rank is 2. This implies that the mechanism has redundant gain in linear velocity. The eigenvalues based on the constraint equation based approach predicts gain in angular velocity, which is not true.

4 Conclusions

In this paper we have developed a modified Jacobian formulation which can determine instantaneous redundant gain in DOF. This is enumerated through numerical examples, we have also emphasised on the limitations of existing approach through these examples.



(a) Actual Manipulator (b) Equalised Representation

Figure 6: Manipulator for larger workspace

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