Design of Force-amplifying Compliant Mechanisms for Resonant Accelerometers

Shyamsananth Madhavan, G. K. Ananthasuresh

Abstract

In this work, we present the design of the Force-amplifying Compliant Mechanisms (FaCMs) that are integrated into micromachined resonant accelerometers to increase their sensitivity. An FaCM is a mechanical amplifier that utilizes the force applied at one point to give an amplified force at another point in an elastic continuum. The inertial force acting on the accelerometer device causes a shift in the resonant frequency of the sensing beams by exerting an axial load on the beams. The role of an FaCM is to amplify this axial force and thus enhance the frequency-shift in the beams. In this work, we present FaCMs that are synthesized using topology optimization procedure for compliant mechanisms. A comparison of these FaCMs with existing force-amplifier lever mechanism is presented. One of the FaCMs outperforms the lever by a factor of six. In order to further improve the designs of the FaCM, we use the selection-map technique. Selection-map is a newly developed method used in place of shape and size optimization for re-designing single-input-single-output (SISO) compliant mechanisms as per user specifications. The evaluations of the re-designed FaCMs indicate better performance as compared to the original one.

Keywords: Compliant mechanisms, Topology Optimization, Selection map, Resonant accelerometer.

1 Introduction

A compliant mechanism is a joint-free, monolithic mechanism comprising flexible segments that deliver motion in desired directions based on its elastic deformation. Compliant mechanisms can be used for mechanical amplification in micromachined inertial sensors. There are two classes of mechanical amplifiers, namely Displacement-amplifying Compliant Mechanism (DaCM) [1] and Force-amplifying Compliant Mechanism (FaCM) [2]. The functionality of a DaCM is to utilize the force or displacement applied at one point to produce an amplified displacement at another point in an elastic continuum and thus enhancing the geometric advantage of the mechanism. In contrast to DaCM, an FaCM utilizes the force or displacement applied at one point to provide an amplified force at another point in an elastic continuum and thus increasing the mechanical advantage of the mechanism. These amplifiers are equivalent to the mechanical levers in functionality. But unlike in a mechanical lever with joints, there is stiffness associated with compliant mechanisms that plays an important role in motion or force transfer.

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1.1 Role of an FaCM in resonant accelerometers

A resonant accelerometer consists of a proof-mass attached to the sensing element called the *resonator beam.* The resonating beams are electrostatically actuated at resonance by applying a sinusoidal voltage. An external inertial acceleration exerts an axial load on the sensing beams and thus creates a resonant frequency-shift in them. The sensitivity of the sensor is the frequency-shift occurred in it. This sensitivity of a sensor can be enhanced with either electronic or mechanical amplifiers. The total noise-floor (output of the sensor in the absence of any signal) that depends on the total noise present in both the mechanical and the electronic components is dominated by the electronic ones in the sensor [3]. Therefore, it makes sense to amplify the signal by mechanical means, the noise corresponding to which is lower in comparison.

We use the mechanical FaCMs to increase the axial force transmitted to the resonating beam considering the functionality and advantages. Figure 1 schematically shows the role of an FaCM in a resonant accelerometer. An FaCM is attached between the proof-mass and the resonating beam-pair. Its main purpose is to amplify the force experienced at its interface point with the proof-mass and deliver the amplified force to the resonating beam in the axial direction. The force-amplification in this case is based on the stiffness parameters of the proof-mass suspensions, resonating beams, and intrinsic stiffness of the mechanism. The earliest model of force-amplifier in resonant sensing application was a mechanical lever of second type [4]. The term FaCM was first coined in [2], where optimal designs of mechanical levers were used.

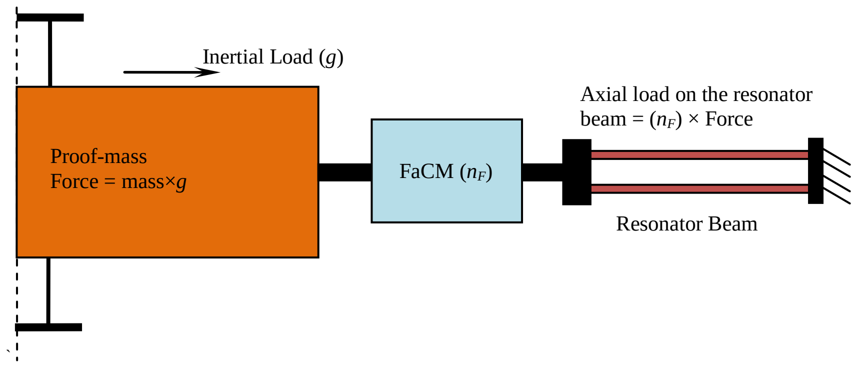


Figure 1: Right symmetric half depicting the coupling of an FaCM to the resonant accelerometer.

This paper is based on our previous work in [5] that presented the analysis methods used for the computation of the frequency-shift when an FaCM is integrated with the accelerometer device. In this work, the focus is on the design methods of an FaCM. This paper comprises three sections. In Section 2, the synthesis of the FaCMs using topology optimization method, the objective function and its sensitivity analysis are described. Also, the comparison of FaCMs based on the amplification factor and functionality is presented. The shape and size modification in FaCM using the re-designing selection-map tool is discussed in Section 3. Conclusions from the work are presented in Section 4.

2 Synthesis of an FaCM

The design of the FaCMs is based on our previous work on DaCMs [1] and methods reported in the literature [2]. The topology optimization method is used for synthesizing the design of compliant mechanisms. Topology optimization refers to the optimum distribution of materials within a prescribed domain. An FaCM has different stiffness at its input and output ends. These stiffness parameters are included in the objective function. The primary objective function used in this work is based on formulation given in [2]. The optimization problem is posed as in Eq. (1) and the sensitivity analysis of the objective function is carried out using the *adjoint* method described in Eq. (2). The continuous optimization method with Solid Isotropic Material with Penalisation (SIMP) interpolation is used in this work.

Objective function:

$$\underset{\rho}{\text{Max}}\ A = \underset{\rho}{\text{Max}} \left(\frac{F_{out}}{F_{in}}\right) = \underset{\rho}{\text{Max}} \left(\frac{k_{out}\mathbf{U}_{out}}{F_{in}}\right) = \frac{k_{out}}{F_{in}}\ \text{Max}\left(\mathbf{U}_{out}\right) \tag{1}$$

$$\Lambda : v^T \rho - v_{specified} \leq 0,\ \mathbf{KU_{out}} = \mathbf{F_{out}},\ \mathbf{KU_{in}} = \mathbf{F_{in}}$$

Sensitivity Analysis:

$$\mathbf{K} = \rho^{penal}\mathbf{k}_0$$

$$\frac{\partial \mathbf{k}}{\partial \rho} = penal(\rho)^{penal-1}\mathbf{k}_0 \tag{2}$$

$$\frac{\partial(\mathbf{U}_{out})}{\partial \rho} = -penal(\rho)^{penal-1}\mathbf{U}_{out}{}^T\mathbf{k}_0\mathbf{U}_{in}$$

where, F_{in} is the force applied at the input port, k_{out} is a spring incorporated at the output port, $\mathbf{U_{out}}$ is the resulting output displacement, $\mathbf{U_{in}}$ is the input displacement, **K** is the effective stiffness matrix of the finite element model and the output spring, **u** is the global displacement vector, **ρ** is the design variables, *penal* is the penalization factor, $\boldsymbol{k_o}$ is the element stiffness matrix and v is the volume of the domain. The shape and size of the FaCMs are modified and interfaced with the resonant accelerometer as shown in Fig. 2(a)-(b) along with the lever mechanism.

These mechanisms are evaluated based on their intrinsic amplification factor (n_F) and capability to produce frequency shift in the resonating beams. The amplification factor is determined using *bloc-force* method, where force is applied on the input port, while constraining the output port as shown in Fig. 2(c). The amplification factor is determined as $n_F = F_{reation} / F_{in}$. The sensitivity of the beams (Δf) is evaluated as difference between resonance frequency with axial loading (f_l) and resonance frequency without loading (f_0). The methods used for performance evaluation of the FaCMs are elaborately explained in [5]. The performance estimates of FaCMs and lever are tabulated in Table 1.

Table 1: Performance estimates of FaCMs and Lever

Parameters	***FaCM1***	***FaCM2***	***FaCM3***	***FaCM4***	***Lever***
n_F	1.45	0.83	**26**	1.74	**4.3**
Δf (Hz)	4	1	**37**	4.2	**6.2**

Based on n_F and Δf, we can observe that FaCM3 outperforms the Lever by a factor of six. These mechanisms are micromachined and integrated into accelerometer device.

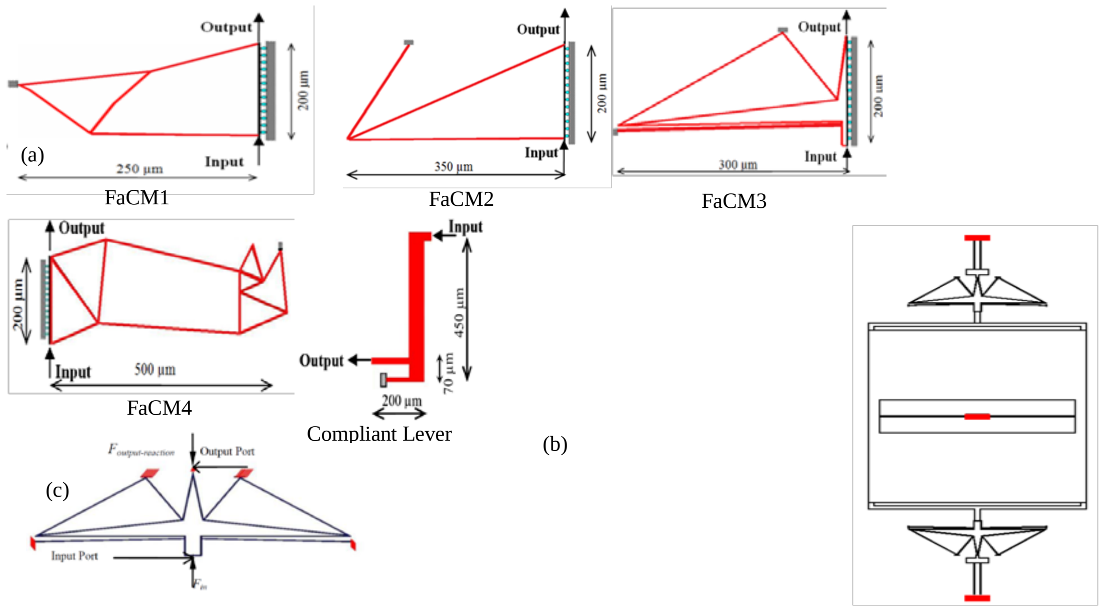


Figure 2: (a) Designed FaCMs, (b) Resonant Accelerometer with FaCM integrated, (c) Boundary conditions for Bloc-force method

3 Re-design of an FaCM using Selection Map Technique

The selection-map technique is an interactive technique used for designing single-input-single-output (SISO) compliant mechanisms that meet the practical requirements of the users [6]. The tool is developed as a GUI, which includes selection panels and a plot area. The user-specifications include minimum and maximum values of the input force F_{in}, input displacement x_{in}, output load F_{out}, output displacement x_{out}, stiffness at input k_{in} and external stiffness at the output k_{ext}. The re-design of the FaCM will include the change in its in-plane width, thickness, stretch along x-y directions, or material, while the same topology will be retained. An FaCM in the resonant accelerometer with the aforementioned specifications is illustrated in Fig. 3. The proof-mass suspension is modelled as the stiffness at the input, while the resonator will exert the stiffness at the output of the mechanism. The input force and displacement are considered at the input port (i.e., the interface of the FaCM with the proof-mass) and similarly the output parameters are considered at the interface of the FaCM and the resonator. In the case of resonant sensing, the user-specifications will be in terms of sensitivity and base frequency of the sensor beam, as these are the major design parameters for the resonant application. In order to use available selection-map technique, we transform the resonant-sensor user-specifications to the existing user-specifications. The procedure used for determination of output force specification from sensitivity and base frequency is outlined.

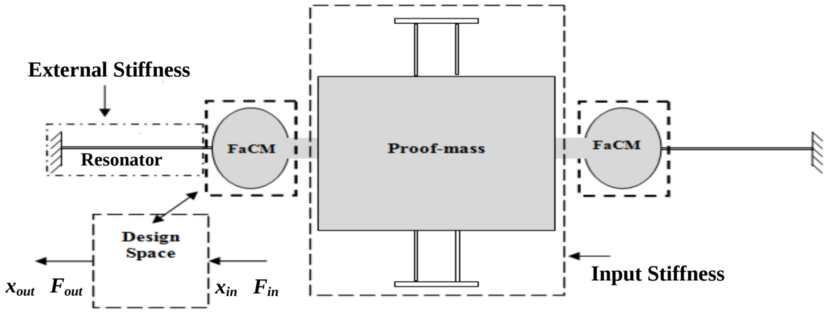


Figure 3: Identification of the design space and the user specifications in a resonant accelerometer device

The sensor frequency parameters are used to find the corresponding output force F_{out} of the compliant mechanism. A user-specification variable **'C'** is defined. This variable is mathematically represented in Eq. (3). The loaded frequency f_l is calculated based on specified f_0 and **C** as $f_l = f_0(1+\mathbf{C}g)$. Equation (4) describes the characteristic eigenfrequency equation for an axially-stiffened and end-restrained beam.

$$C = \left(\frac{f_l - f_0}{f_0} \right) \Big/ g \tag{3}$$

$$T^2 \sin(b)\sinh(a) - 2ab\cos(b)\cosh(a) + 2ab = 0 \tag{4}$$

$$a = \frac{1}{2}\sqrt{2T^2 + 2\sqrt{T^4 + 4\lambda_l^4}} \qquad b = \frac{1}{2}\sqrt{-2T^2 + 2\sqrt{T^4 + 4\lambda_l^4}} \tag{5}$$

The terms in Eq. (4) consists of just two variables namely, non-dimensional load variable T and non-dimensional frequency variable λ_l, where $\lambda_l^4 = \left(4\pi^2 \rho A l^4 / EI\right) f_l^2$ and $T^2 = F_{out} l^2 / EI$. On substituting f_l in Eq. (4) and solving it iteratively, the value of output force is determined from T. Based on this procedure, the user-specifications used in the design cases are taken as follows. The single beam resonator of dimensions (180 μm × 4 μm × 25 μm) with f_0 value around 1 MHz and k_{ext} of 86000 N/m is considered. The values of **C** determined are 4e-6 and 6e-6. The output force for the corresponding **C** variable is determined to be 2μN and 3μN for frequency shift ranging from 40 Hz and 60 Hz. The FaCM3 is considered for re-design in the following cases.

3.1 Design case-1

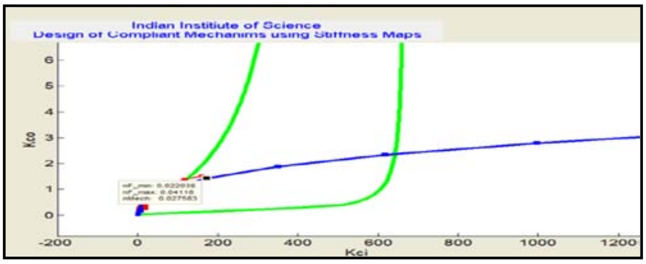
The user-specifications are tabulated in Table 2. The maximum and minimum values corresponding to external spring stiffness are considered zero. The feasible map for the above user-specifications lies in the positive quadrant of the SL parameters k_{ci}-k_{co} axes as shown in Fig. 4(a). The FaCM3 mechanism lies inside the feasible map, thus meeting the user's requirements. The mechanism is re-designed using the parameter curves. The parameter curves correspond to resizing in *x*, resizing in *y*, and resizing the in-plane width and thickness of the mechanism. Figures 4(a) indicate the curves emerging from the mechanism.

Table 2: User Specifications for Design Cases

Specification Variables	***Design-1***		***Design-2***	
	Min	*Max*	*Min*	*Max*
F_{in}(N)	4e-7	6e-7	4e-7	6e-7
x_{in}(m)	8e-10	10e-9	8e-10	10e-9
F_{out}(N)	2e-6	3e-6	2e-6	3e-6
x_{out}(m)	1e-11	1.2e-11	1e-11	1.2e-11
K_{in}(N/m)	50	50	500	500
K_{out}(N/m)	0	0	0	0

In this case, the in-plane width re-design parameter is selected as the parameter curve moves deep into the feasible space. Using the data cursor, the dot is moved along the curve and corresponding change in width as well as the amplification factor (*n*), n_{min} and n_{max} are observed.

a)



b)

Figure 4: (a) Re-design using the parameter curves, (b) FaCM with modified in-plane width

The point on the curve where the *n* lies between n_{min} and n_{max} indicates that the mechanism is realistic and satisfies the requirements. It is important to note that although the parameter curve led into the feasible space, the criterion of *n* being within the n_{min} and n_{max} range needs to be met for the practically utility of the mechanism. Such a point is located on the curve and the in-plane width is progressively increased from its current width of 2.5 μm to 3.4 μm. The re-designed FaCM with increased in-plane width along with the original one is illustrated in Fig. 4(b).

3.2 Design case-2

The user-specifications are tabulated in Table 2. The input stiffness is increased ten times that of the previous case. The remaining specifications are the same as in the previous case. The feasible map for the above user-specifications lies in the positive quadrant of the SL parameters k_{ci}-k_{co} axes, which is shown in Fig. 5(a), thus meeting the user requirements. It can be seen that the area of the feasible map when compared to the previous case, changes according to the given specifications. In this case, the re-design parameter selected was the stretch in the *x*-direction. Although most of the parameter curves lead into the feasible map which is evident in the figure, only this parameter curve was found feasible considering the *n*-criterion. A point on the resize in the *x*-parameter curve indicated *n* to be well within n_{min} and n_{max} limits. The mechanism is stretched by a factor of 1.254 along *x*-plane. The re-designed FaCM plotted in dotted lines along with the original one is illustrated in Fig. 5(b).

a)

b)

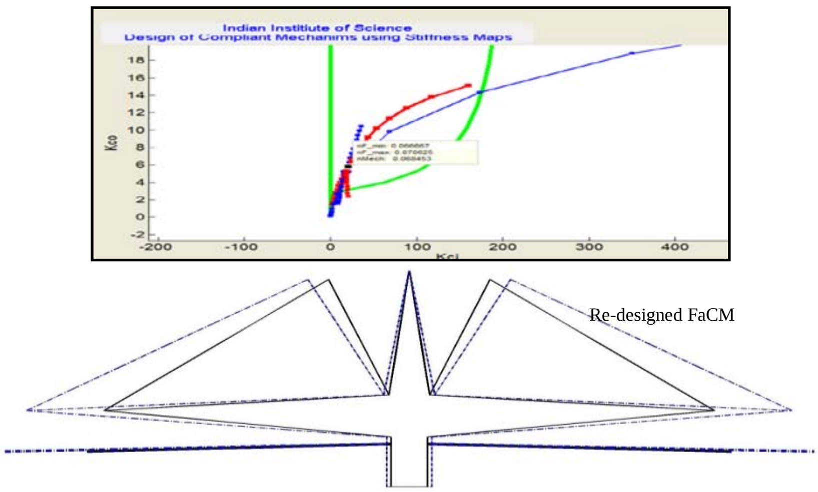


Figure 5: (a) Re-design using the parameter curves, (b) FaCM stretched in *x*-axis

3.3 Performance of re-designed FaCMs

The modified FaCMs are modelled in commercial software AUTOCADD and integrated with the accelerometer device. Simulations are carried out to observe the enhancement in the sensitivity of the resonator as well as the amplification factor. The amplification factor, base and loaded frequency along with the frequency-shift are tabulated in Table 3. The frequency shift is enhanced up to two times by the modified FaCMs and its value is within the user-specified range. The range specified by the user was 40-60 Hz and the obtained shift is within this range. These design cases indicate the effectiveness of the selection technique for the given user-inputs.

Table 3: Comparison of Re-designed FaCMs with original FaCM3

Device with FaCM	n_F	Δf (Hz)
Original FaCM	**26**	**37**
Case-1: In-plane Width	**28.17**	**48**
Case-2: Stretched in *x*-direction	**27.32**	**44**

4 Conclusions

In this paper, the need of FaCM in resonant sensing, well-defined design methods and re-design method are presented. The synthesis of the mechanism using topology optimization and further shape and size modification gave rise to FaCM3 that outperforms the existing Lever by a factor of six. The re-design of the FaCM using the selection-map tool for a given set of user specifications provides further enhancement in the frequency-shift. The concatenation of FaCMs with lever providing multi-stage amplification will be considered in future work.

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