An Improved Dynamic Modeling of a 3-RPS Parallel Manipulator using the concept of DeNOC Matrices

A. Rahmani Hanzaki, E. Yoosefi

Abstract

A recursive dynamic modeling of a three-DOF parallel robot, namely, three-Revolute-Prismatic-Spherical (3-RPS) parallel manipulator is reported in this work. Euler parameters are utilized to define the 3-DOF orientation of its moving platform, because the coordinates are free of singularity. For the dynamic modeling, the concept of the Decoupled Natural Orthogonal complement (DeNOC) matrices is employed. The necessity of the concept is to use a set of independent coordinates, while Euler parameters are not. Hence, the improved recursive dynamic modeling reported somewhere else is utilized. This is especially useful for forward dynamic modeling. Finally, the results obtained are compared with those achieved from ADAMS model to validate them.

Keywords: 3-RPS manipulator; Euler parameters; Dynamic modeling, DeNOC matrices;

1 Introduction

Parallel manipulators have more advantages over serial chains. Some of the advantages are that they have higher speed, higher stiffness, and larger load capacity. On the other hand, they have special usages, which serial robot are not be able to accomplish. Therefore, in the last decades, many researchers have been working on kinematics and dynamics of parallel structure mechanisms. Dynamic analysis plays an important role in predicting the behavior of such mechanical systems and achieving their best performance.

There are many different approaches to derive the equations of motion for mechanisms. Two popular approaches used for dynamic modeling are (i) Newton-Euler (NE) formulation and (ii) Euler-Lagrange (EL) formulation [1]. The NE formulation requires the equations of motion to be written for each body of manipulator hence leading to a large number of equations; therefore, it is not useful for large systems. The EL formulation allows the elimination of all reaction forces and moments, so it is a simplified approach for simulation but requires complex partial derivatives. Alternatively, there are several methods that start with NE formulation and end up with EL equations. One such methodology uses the Decoupled Natural Orthogonal Complement (DeNOC) matrices [2, 3]. One of the major advantages of using the DeNOC matrices is the availability of recursive dynamics even for general closed- loop systems [4, 5].

Three-RPS mechanism shown in Fig. (1), is one of the lower-mobility parallel manipulators, whose DOF is three. The manipulator has three legs, while each one is a serial kinematic chain consisting of a revolute joint (R), a prismatic joint (P), and a

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spherical joint (S). Three-RPS parallel manipulator was proposed by Hunt, [6], for the first time. Lee and Shah, [7], analyzed the inverse and the forward kinematics of the manipulator. The velocity and the acceleration relationship between the independent output motions and three input parameters in close form of the mechanism were established by Fang and Huang in [8]. Huang in [9] presented a method based on the screw theory to determine the possible motion characteristics for 3-RPS mechanism. [10] Investigated the kinematic of the 3-RPS parallel manipulator that its platform orientation described in terms of Euler angles.

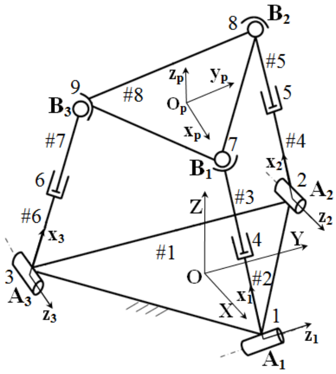


Figure 1: Spatial 3-RPS parallel manipulator

Several methods have been applied to formulate the dynamic of 3-RPS parallel manipulator. [11 and 12] used Lagrange formulation to derive the equations of motion of the manipulator. [13, 14] presented a dynamic modeling based on virtual work principle.

In this paper, dynamic analysis of the 3-RPS parallel manipulator using DeNOC matrices is presented, while the relative orientations due to spherical joints are represented in terms of relative Euler parameters. It is well-known that using the Euler parameters to define orientation of a moving coordinate system with respect to (w.r.t) another one is advantageous, because there is no inherent geometrical singularity in their equations [15]. Hence, in [5], kinematic and dynamic modeling of a multibody system having spherical joints was described using Euler parameters. The authors solved the kinematics of the same manipulator in [16].

2 Equations of Motion

In this section, some definitions and concepts associated with the dynamic formulation of an n-link open chain serial system using the Decoupled Natural Orthogonal Complement (DeNOC) matrices is explained.

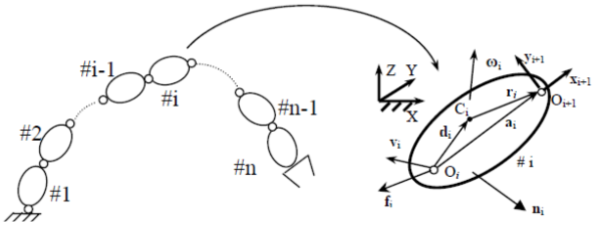


Figure 2: An n-link serial manipulator

Figure (2) shows the i^{th} link of an n-link serial chain. O_i is the origin of coordinate system, which is attached to the i^{th} link at the joint connected the i^{th} and the $(i\text{-}1)^{st}$ links. The mass center of the i^{th} link is considered at C_i. Further, the 3-

dimensional position vectors $\mathbf{d}_i$ from O_i to the mass center of the i[th] link, and $\mathbf{r}_i$ from the mass center of the i[th] link to O_{i+1} are defined.
The 6-dimensional twist and wrench vector associated with the i[th] link are also identified as

$$\mathbf{t}_i = \begin{bmatrix}\boldsymbol{\omega}_i\\ \mathbf{v}_i\end{bmatrix} \quad \text{and} \quad \mathbf{w}_i = \begin{bmatrix}\mathbf{n}_i\\ \mathbf{f}_i\end{bmatrix} \tag{1}$$

where $\boldsymbol{\omega}_i$ and $\mathbf{v}_i$ are the 3-dimensional vectors of angular velocity and linear velocity of point O_i, respectively, while $\mathbf{n}_i$ and $\mathbf{f}_i$ are the 3-dimensional vectors of resultant moment about O_i, and the resultant force at O_i, in that order. Note that the twist of the i[th] link can be written recursively in terms of the twist of link i-1, as

$$\mathbf{t}_i = \mathbf{A}_{i,i-1}\mathbf{t}_{i-1} + \mathbf{p}_i\dot{\boldsymbol{\theta}}_i \tag{2}$$

where

$$\mathbf{A}_{i,i-1} = \begin{bmatrix}\mathbf{1} & \mathbf{O}\\ \tilde{\mathbf{a}}_{i,i-1} & \mathbf{1}\end{bmatrix};\ \mathbf{p}_i = \begin{bmatrix}\mathbf{u}_i\\ \mathbf{0}\end{bmatrix} \text{ for revolute; } \mathbf{p}_i = \begin{bmatrix}\mathbf{0}\\ \mathbf{u}_i\end{bmatrix} \text{ for prismatic joints} \tag{3}$$

In Eq. (3), $\tilde{\mathbf{a}}_{i,i-1}$ is the 3×3 cross-product matrix associated with the vector $\mathbf{a}_{i,i-1}$, which defines position O_{i-1} from O_i. Moreover, $\mathbf{O}$ and $\mathbf{1}$ are the 3×3 zero and identity matrices, respectively, whereas, $\mathbf{0}$ is the 3-dimensional vector of zeros. Furthermore, $\mathbf{u}_i$ is 3-dimensional unit vector parallel to the i[th] joint axis.

If the i[th] link connected to the (i-1)[st] link by a spherical joint, where Euler parameters are used define the rotation, Eq. (2) can be written as

$$\mathbf{t}_i = \mathbf{A}_{i,i-1}\mathbf{t}_{i-1} + \mathbf{P}_i\dot{\mathbf{e}}_i \tag{4}$$

while $\mathbf{P}_i$ is 6×3 joint-motion propagation matrix that is defined as

$$\mathbf{P}_i = \begin{bmatrix}\mathbf{G}_i^*\\ \mathbf{O}\end{bmatrix};\text{ where } \mathbf{G}_i^* = \left[\mathbf{R}_{i,i-1} + \mathbf{1}\right]/e_{0i} \tag{5}$$

in which $\mathbf{R}_{i,i-1}$ is the rotation matrix that transform a vector from the frame connected to the i[th] body into the frame connected to the (i-1)[th] body.

The generalized twist of the entire system for the n rigid bodies in the system is written as

$$\mathbf{t} = \mathbf{N}\dot{\boldsymbol{\theta}} \text{ where } \mathbf{N} = \mathbf{N}_l\mathbf{N}_d \tag{6}$$

Matrices $\mathbf{N}_l$ and $\mathbf{N}_d$, are the 6n×6n lower triangular matrix, and the 6n×(r+3s) diagonal matrix, respectively, where r and s represent the number of one-DOF revolute/prismatic joints and three-DOF spherical joints, respectively. Hence, $\mathbf{N}_l$ and $\mathbf{N}_d$ are defined by

$$\mathbf{N}_l = \begin{bmatrix} 1 & & & & & \\ \mathbf{A}_{21} & 1 & & & & \\ \vdots & & \ddots & & & \\ \mathbf{A}_{i,1} & \cdots & \mathbf{A}_{i,i-1} & 1 & & \\ \vdots & & & & \ddots & \\ \mathbf{A}_{n,1} & & \cdots & & \mathbf{A}_{n,n-1} & 1 \end{bmatrix};\ \mathbf{N}_d = \begin{bmatrix} \mathbf{P}_1 & & & & \\ & \mathbf{P}_2 & & & \\ & & \ddots & & \\ & & & \mathbf{P}_i & & \\ & & & & \ddots & \\ & & & & & \mathbf{P}_n \end{bmatrix} \tag{7}$$

where $\mathbf{P}_i$ is the joint-motion propagation matrix for the three-DOF spherical joints obtained from Eq. (5), or the joint-motion propagation vector for the one-DOF revolute/prismatic joints, $\mathbf{p}_i$ as presented in Eq. (3), depending on the i[th] link connected to its previous one by a three-DOF joint or a one-DOF joint. Accordingly, $\dot{\boldsymbol{\theta}}$ of Eq. (6), is defined as the (r+3s)-dimensional vector of independent generalized speeds, which contains $\dot{\theta}$s for revolute/prismatic joints associated to vector $\mathbf{p}$ in matrix $\mathbf{N}_d$, and $\dot{\mathbf{e}}$ s for spherical joints corresponding to $\mathbf{P}$ elements in matrix $\mathbf{N}_d$.
For the i[th] link, the Newton-Euler equations of motion are expressed as

$$\mathbf{M}_i\dot{\mathbf{t}}_i + \mathbf{W}_i\mathbf{M}_i\mathbf{E}_i\mathbf{t}_i = \mathbf{w}_i \tag{8}$$

where the 6×6 matrices, $\mathbf{M}_i$, $\mathbf{W}_i$, and $\mathbf{E}_i$, are given by

$$\mathbf{M}_i = \begin{bmatrix}\mathbf{I}_i & m_i\tilde{\mathbf{d}}_i\\ -m_i\tilde{\mathbf{d}}_i & m_i\mathbf{1}\end{bmatrix};\quad \mathbf{W}_i = \begin{bmatrix}\tilde{\boldsymbol{\omega}}_i & \mathbf{0}\\ \mathbf{0} & \tilde{\boldsymbol{\omega}}_i\end{bmatrix};\quad \mathbf{E}_i = \begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix} \tag{9}$$

in which $\mathbf{I}_i$ is the 3×3 inertia tensor for the ith link about O_i, and m_i is its mass. For a multibody system with n rigid links, NE equations of motion are written as

$$\mathbf{M}\dot{\mathbf{t}} + \mathbf{WMEt} = \mathbf{w} \qquad (10)$$

where the 6n×6n matrices, $\mathbf{M}$, $\mathbf{W}$, $\mathbf{E}$, and the 6n-dimensional vectors, $\mathbf{t}$ and $\mathbf{w}$, are defined as follows

$$\mathbf{M} = \mathrm{diag}[\mathbf{M}_1, \mathbf{M}_2, \ldots, \mathbf{M}_n]\ ;\ \mathbf{W} = \mathrm{diag}[\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_n]\ ; \mathbf{E} = \mathrm{diag}[\mathbf{E}_1, \mathbf{E}_2, \ldots, \mathbf{E}_n]$$
$$\mathbf{t} = [\mathbf{t}_1^T, \mathbf{t}_2^T, \ldots, \mathbf{t}_n^T]^T \quad ; \quad \mathbf{w} = [\mathbf{w}_1^T, \mathbf{w}_2^T, \ldots, \mathbf{w}_n^T]^T \qquad (11)$$

As shown in [17], if both sides of Eq. (10) are pre-multiplied by $\mathbf{N}^T$, the wrench due to the reaction forces are vanished, and Eq. (10) yield to

$$\mathbf{N}^T(\mathbf{M}\dot{\mathbf{t}} + \mathbf{WMEt}) = \mathbf{N}^T\mathbf{w}^e \qquad (12)$$

For a closed-loop system, by cutting appropriate joints and substituting constraint forces or moments in terms of Lagrange multipliers [5], equations of motion are rewritten as

$$\mathbf{N}^T(\mathbf{M}\dot{\mathbf{t}} + \mathbf{WMEt}) = \mathbf{N}^T(\mathbf{w}^e + \mathbf{w}^\lambda) \qquad (13)$$

in which the associated vectors and matrices are

$$\mathbf{M} = \mathrm{diag}[\mathbf{M}_I, \mathbf{M}_{II}, \ldots, \mathbf{M}_O]\ ;\ \mathbf{W} = \mathrm{diag}[\mathbf{W}_I, \mathbf{W}_{II}, \ldots, \mathbf{W}_O]\ ; \mathbf{E} = \mathrm{diag}[\mathbf{E}_I, \mathbf{E}_{II}, \ldots, \mathbf{E}_O]$$
$$\mathbf{N}_l = \mathrm{diag}[\mathbf{N}_{l\,I}, \mathbf{N}_{l\,II}, \ldots, \mathbf{N}_{l\,O}]\ ;\ \mathbf{N}_d = \mathrm{diag}[\mathbf{N}_{d\,I}, \mathbf{N}_{d\,II}, \ldots, \mathbf{N}_{d\,O}] \qquad (14)$$

and

$$\mathbf{t} = \left[\mathbf{t}_I^T, \mathbf{t}_{II}^T, \ldots, \mathbf{t}_O^T\right]^T \ \ \mathbf{w}^e = \left[\mathbf{w}^{e\,T}_{I}, \mathbf{w}^{e\,T}_{II}, \ldots, \mathbf{w}^{e\,T}_{O}\right]^T;\ \mathbf{w}^\lambda = \left[\mathbf{w}^{\lambda\,T}_{I}, \mathbf{w}^{\lambda\,T}_{II}, \ldots, \mathbf{w}^{\lambda\,T}_{O}\right]^T \qquad (15)$$

Matrices $\mathbf{M}_I$, $\mathbf{M}_{II}$, and $\mathbf{M}_O$ are the generalized mass matrices, as defined in Eq. (11), for the opened subsystems. Other matrices and vectors are similarly defined.

3 Kinematic Modeling

Figure (1) Shows a 3-RPS parallel manipulator, which consists of a moving platform, a fixed base and three extendable legs. Each leg alone is a serial kinematic chain consisting of a revolute joint (R) at the base side and a spherical joint (S) attached to the platform. A prismatic joint (P) allows the lengths of the legs to be changed. The axis of each revolute joint is tangential to the circumscribed circle of the fixed base. In this mechanism, the links are numbered as #1 … #8 and the joints by 1 … 9.

The number of variables required to specify the platform configuration is fourteen, of which, three are independent. Therefore, eleven constraint equations are needed to solve the kinematics of the 3-RPS platform. The kinematic constraint equations are expressed as:

$$\mathbf{\Phi}(\mathbf{q}) = \mathbf{0} \qquad (16)$$

Since, the constraint equations of the 3-RPS parallel manipulator are explained and solved in [16] completely; it is avoided to bring them here again to keep the size of paper reasonable.

4 Dynamic Modeling

The first step for dynamic analysis of the 3-RPS parallel manipulator using DeNOC matrices, as shown in Fig. (3), is opening the closed-loops by cutting joints 8 and 9, which are located at point B_2 and B_3, respectively. Therefore, the system is converted into three serial open chains, namely, #4-#5, #6-#7, and #2-#3-#8, which are considered as subsystems I, II, and III, respectively. Then, the cut joints are substituted with appropriate Lagrange multipliers. Joint 8 is a spherical joint that only have a spatial reaction force, $\mathbf{f}_{85}^\lambda$ whose components along x_5, y_5, and z_5 are f_{85x}^λ, f_{85y}^λ, and f_{85z}^λ , respectively, applied to #5 by #8. Similarly, joint 9 is also a spherical joint that only have a spatial reaction force, $\mathbf{f}_{87}^{\lambda}$ whose components along x_7, y_7, and z_7 are f_{87x}^{λ}, f_{87y}^{λ}, and f_{87z}^{λ} , respectively, applied to #7 by #8.

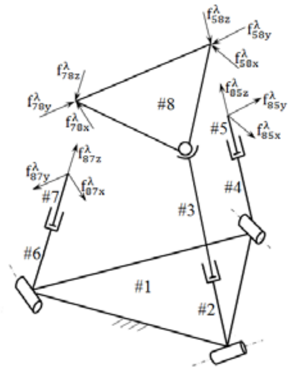


Figure 3: Reaction forces at cut joints 8 and 9

Equations of motion for each open subsystem are written using Eq. (13). The equations of motion for subsystem I are followed as

$$\mathbf{N}_{\mathrm{I}}^{\mathrm{T}}(\mathbf{M}_{\mathrm{I}}\dot{\mathbf{t}}_{\mathrm{I}} + \mathbf{W}_{\mathrm{I}}\mathbf{M}_{\mathrm{I}}\mathbf{E}_{\mathrm{I}}\mathbf{t}_{\mathrm{I}}) = \mathbf{N}_{\mathrm{I}}^{\mathrm{T}}(\mathbf{w}_{\mathrm{I}}^{\mathrm{e}} + \mathbf{w}_{\mathrm{I}}^{\lambda}) \tag{17}$$

where the 12×12 generalized mass matrix, $\mathbf{M}_{\mathrm{I}}$, generalized angular velocity of the subsystem, $\mathbf{W}_{\mathrm{I}}$, and matrix $\mathbf{E}$ are found as

$$\mathbf{M}_{\mathrm{I}} = \mathrm{diag}[\mathbf{M}_4, \mathbf{M}_5]\ ;\ \mathbf{W}_{\mathrm{I}} = \mathrm{diag}[\mathbf{W}_4, \mathbf{W}_5]\ ;\mathbf{E}_{\mathrm{I}} = \mathrm{diag}[\mathbf{E}_4, \mathbf{E}_5] \tag{18}$$

In Eq. (18), $\mathbf{M}_4$ and $\mathbf{M}_5$ are the 6×6 generalized mass matrices of #4 and #5, respectively, while $\mathbf{W}_4$ and $\mathbf{W}_5$ are the 6×6 matrices that depend on angular velocity of #4 and #5, respectively, and $\mathbf{E}_4$ and $\mathbf{E}_5$ are equal and constant, as shown in Eq. (9). The twist associated with the subsystem is defined as:

$$\mathbf{t}_{\mathrm{I}} = [\mathbf{t}_4^{\mathrm{T}} \quad \mathbf{t}_5^{\mathrm{T}}]^{\mathrm{T}} \tag{19}$$

where $\mathbf{t}_4$ and $\mathbf{t}_5$ are the twists of #4 and #5, respectively. In other words, Eq. (19) is obtained as

$$\mathbf{t}_{\mathrm{I}} = \mathbf{N}_{\mathrm{I}}\dot{\boldsymbol{\theta}}_{\mathrm{I}} \tag{20}$$

where $\dot{\boldsymbol{\theta}}_{\mathrm{I}} = [\dot{\theta}_2 \quad \dot{d}_2]^{\mathrm{T}}$ is the vector of independent generalized speeds. Next, matrix $\mathbf{N}$ for subsystem I is obtained by the multiplication of its matrices $\mathbf{N}_{\mathbf{l}}$ and $\mathbf{N}_{\mathbf{d}}$, as follows:

$$\mathbf{N}_{\mathrm{I}} = \mathbf{N}_{\mathrm{l\,I}}\mathbf{N}_{\mathrm{d\,I}} = \begin{bmatrix} \mathbf{p}_2 & \mathbf{0} \\ \mathbf{A}_{54}\mathbf{p}_2 & \mathbf{p}_5 \end{bmatrix} \tag{21}$$

where $\mathbf{p}_2$ and $\mathbf{p}_5$ are the joint propagation vectors for revolute joint 2 and prismatic joint 5 shown in Eq. (3), respectively. Also $\mathbf{A}_{54}$ is the 6×6 twist propagation matrix which transfers the twist of #4 to #5. By neglecting the gravity, the wrench due to external forces is followed as

$$\mathbf{w}_{\mathrm{I}}^{\mathrm{e}} = \left[\mathbf{w}^{\mathrm{e}}{}_4^{\mathrm{T}} \quad \mathbf{w}^{\mathrm{e}}{}_5^{\mathrm{T}}\right]^{\mathrm{T}} \tag{22}$$

where

$$\mathbf{w}_4^{\mathrm{e}} = \mathbf{0} \ \text{ and } \ \mathbf{w}_5^{\mathrm{e}} = \begin{bmatrix} \mathbf{O} \\ \mathbf{R}_{41}\mathbf{R}_{54}[0 \quad 0 \quad f_{p5}]^{\mathrm{T}} \end{bmatrix} \tag{23}$$

In Eq.(23), f_{p5} is the driving force at prismatic joint 5. The wrench due to Lagrange multipliers related to the subsystem is

$$\mathbf{w}_{\mathrm{I}}^{\lambda} = \left[\mathbf{w}^{\lambda}{}_4^{\mathrm{T}} \quad \mathbf{w}^{\lambda}{}_5^{\mathrm{T}}\right]^{\mathrm{T}} \quad \text{where} \quad \mathbf{w}_4^{\lambda} = \mathbf{0} \ \text{ and } \ \mathbf{w}_5^{\lambda} = \mathbf{A}_{58}'\mathbf{w}_8^{\lambda} \tag{24}$$

$\mathbf{w}_8^{\lambda}$ is the wrench due to Lagrange multipliers at joint 8, i.e.,

$$\mathbf{w}_8^{\lambda} = \begin{bmatrix} \mathbf{O} \\ \mathbf{R}_{41}\mathbf{R}_{54}[\mathbf{f}_{85}^{\lambda}] \end{bmatrix} \ \text{ and } \ [\mathbf{f}_{85}^{\lambda}] = [f_{85x}^{\lambda} \quad f_{85y}^{\lambda} \quad f_{85z}^{\lambda}]^{\mathrm{T}} \tag{25}$$

Similar to the equations of motion for subsystem I, the same for subsystem II are written while links #4 and #5 are substituted by links #6 and #7, respectively. It is avoided to explain it here again since both subsystems are completely comparable.

The final subsystem is III, whose equations of motion are written similar to Eq. (17), where $\mathbf{M}_{\mathrm{III}}$, $\mathbf{W}_{\mathrm{III}}$, and $\mathbf{E}_{\mathrm{III}}$ are defined like Eq. (18). The twist associated with the subsystem is defined as

$$\mathbf{t}_{\mathrm{III}} = [\mathbf{t}_2^{\mathrm{T}} \quad \mathbf{t}_3^{\mathrm{T}} \quad \mathbf{t}_8^{\mathrm{T}}]^{\mathrm{T}} \tag{26}$$

which be obtained as

$$\mathbf{t}_{\mathrm{III}} = \mathbf{N}_{\mathrm{III}}\dot{\boldsymbol{\theta}}_{\mathrm{III}} \text{ where } \dot{\boldsymbol{\theta}}_{\mathrm{III}} = \begin{bmatrix}\dot{\theta}_1 & \dot{\mathrm{d}}_1 & \dot{\mathbf{e}}_3\end{bmatrix}^{\mathrm{T}} \tag{27}$$

Matrix $\mathbf{N}_{\mathrm{III}}$ is identified as

$$\mathbf{N}_{\mathrm{III}} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \mathbf{O} \\ \mathbf{A}_{32}\mathbf{p}_1 & \mathbf{p}_4 & \mathbf{O} \\ \mathbf{A}_{82}\mathbf{p}_1 & \mathbf{A}_{83}\mathbf{p}_4 & \mathbf{P}_7 \end{bmatrix} \tag{28}$$

where $\mathbf{p}_1$ and $\mathbf{p}_4$ are the joint propagation vectors of the revolute joint 1 and prismatic joint 4, respectively, and $\mathbf{P}_7$ is the joint propagation matrix of the spherical joint 7, as given by

$$\mathbf{P}_7 = \begin{bmatrix}\mathbf{R}_{21}\mathbf{G}_7^* \\ \mathbf{O}\end{bmatrix}; \text{where } \mathbf{G}_7^* = [\mathbf{R}_{83} + \mathbf{1}]/\mathrm{e}_{30} \tag{29}$$

The wrenches due to external forces and Lagrange multipliers are

$$\mathbf{w}_{\mathrm{III}}^{\mathrm{e}} = \begin{bmatrix}\mathbf{w}^{\mathrm{e}\,\mathrm{T}}_2 & \mathbf{w}^{\mathrm{e}\,\mathrm{T}}_3 & \mathbf{w}^{\mathrm{e}\,\mathrm{T}}_8\end{bmatrix}^{\mathrm{T}}, \text{where}$$

$$\mathbf{w}_2^{\mathrm{e}} = \mathbf{0} \text{ and } \mathbf{w}_3^{\mathrm{e}} = \begin{bmatrix}\mathbf{O} \\ \mathbf{R}_{21}\mathbf{R}_{32}[0 \quad 0 \quad \mathrm{f}_{\mathrm{p}4}]^{\mathrm{T}}\end{bmatrix} \text{ and } \mathbf{w}_8^{\mathrm{e}} = \mathbf{0} \tag{30}$$

and

$$\mathbf{w}_{\mathrm{III}}^{\lambda} = \begin{bmatrix}\mathbf{w}^{\lambda\,\mathrm{T}}_2 & \mathbf{w}^{\lambda\,\mathrm{T}}_3 & \mathbf{w}^{\lambda\,\mathrm{T}}_8\end{bmatrix}^{\mathrm{T}}, \text{in which}$$

$$\mathbf{w}_2^{\lambda} = \mathbf{w}_3^{\lambda} = \mathbf{0} \text{ and } \mathbf{w}_8^{\lambda} = -\mathbf{A}'_{78}\mathbf{w}_8^{\lambda} - \mathbf{A}'_{79}\mathbf{w}_9^{\lambda} \tag{31}$$

In Eq. (31), $\mathbf{w}_8^{\lambda}$ and $\mathbf{w}_9^{\lambda}$ are already defined. Now, the equations of motion for the whole system can be written as

$$\mathbf{N}^{\mathrm{T}}(\mathbf{M}\dot{\mathbf{t}} + \mathbf{WMEt}) = \mathbf{N}^{\mathrm{T}}(\mathbf{w}^{\mathrm{e}} + \mathbf{w}^{\lambda}) \tag{32}$$

in which the associated matrices and vectors in Eq. (32) are as follows

$$\mathbf{M} = \mathrm{diag}[\mathbf{M}_{\mathrm{I}}, \mathbf{M}_{\mathrm{II}}, \mathbf{M}_{\mathrm{III}}]\,;\ \mathbf{W} = \mathrm{diag}[\mathbf{W}_{\mathrm{I}}, \mathbf{W}_{\mathrm{II}}, \mathbf{W}_{\mathrm{III}}]\,;\ \mathbf{E} = \mathrm{diag}[\mathbf{E}_{\mathrm{I}}, \mathbf{E}_{\mathrm{II}}, \mathbf{E}_{\mathrm{III}}]$$

$$\mathbf{N} = \mathrm{diag}[\mathbf{N}_{\mathrm{I}}, \mathbf{N}_{\mathrm{II}}, \mathbf{N}_{\mathrm{III}}] \tag{33}$$

and

$$\mathbf{t} = \begin{bmatrix}\mathbf{t}_{\mathrm{I}}^{\mathrm{T}}, \mathbf{t}_{\mathrm{II}}^{\mathrm{T}}, \mathbf{t}_{\mathrm{III}}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \ \mathbf{w}^{\mathrm{e}} = \begin{bmatrix}\mathbf{w}^{\mathrm{e}\,\mathrm{T}}_{\mathrm{I}}, \mathbf{w}^{\mathrm{e}\,\mathrm{T}}_{\mathrm{II}}, \mathbf{w}^{\mathrm{e}\,\mathrm{T}}_{\mathrm{III}}\end{bmatrix}^{\mathrm{T}};\ \mathbf{w}^{\lambda} = \begin{bmatrix}\mathbf{w}^{\lambda\,\mathrm{T}}_{\mathrm{I}}, \mathbf{w}^{\lambda\,\mathrm{T}}_{\mathrm{II}}, \mathbf{w}^{\lambda\,\mathrm{T}}_{\mathrm{III}}\end{bmatrix}^{\mathrm{T}} \tag{34}$$

The equations of motion for the 3-RPS parallel manipulator, i.e. Eq. (32), can be used to obtain the Lagrange multipliers and the three driving forces at the prismatic joints that indicate in Fig. (4). To illustrate dynamic analysis of the manipulator, the system parameters used for computation are given as:

$$\mathrm{m}_2 = \mathrm{m}_4 = \mathrm{m}_6 = 0.123\mathrm{kg}, \quad \mathrm{m}_3 = \mathrm{m}_5 = \mathrm{m}_7 = 0.392\mathrm{kg}, \quad \mathrm{m}_8 = 1.013\mathrm{kg}$$

$$\mathbf{I}_2 = \mathbf{I}_4 = \mathbf{I}_6 = \mathrm{diag}[1.53 \quad 1.63\times10^3 \quad 1.63\times10^3]$$

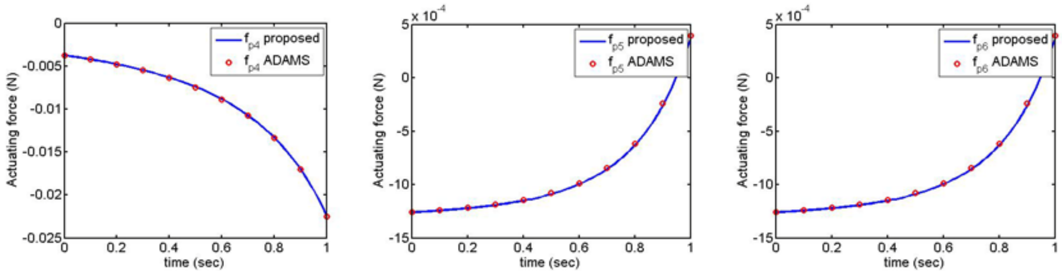
$$\mathbf{I}_3 = \mathbf{I}_5 = \mathbf{I}_7 = \mathrm{diag}[3.36\times10^3 \quad 3.36\times10^3 \quad 19.61]$$

$$\mathbf{I}_8 = \mathrm{diag}[1.28\times10^3 \quad 11.41\times10^3 \quad 12.67\times10^3]$$

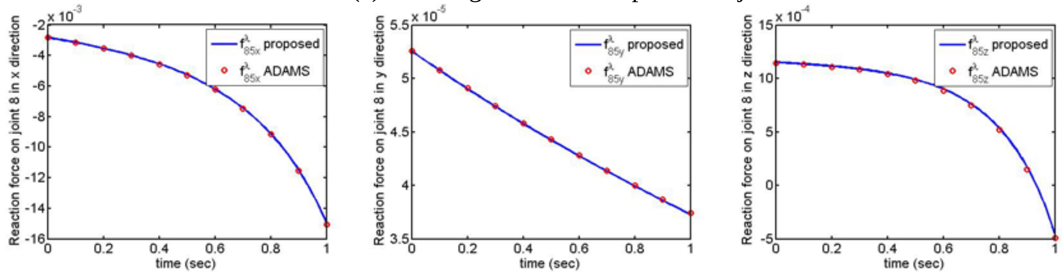
$$\mathbf{d}_1 = 10\frac{\pi}{3}\mathrm{time}\,;\quad \mathbf{d}_2 = \mathbf{d}_3 = -10\frac{\pi}{3}\mathrm{time}$$

where m_i and $\mathbf{I}_i$ are mass and inertia matrix of the i^{th} link, respectively, and $\mathbf{d}_i$ is actuator input of i^{th} prismatic joint.

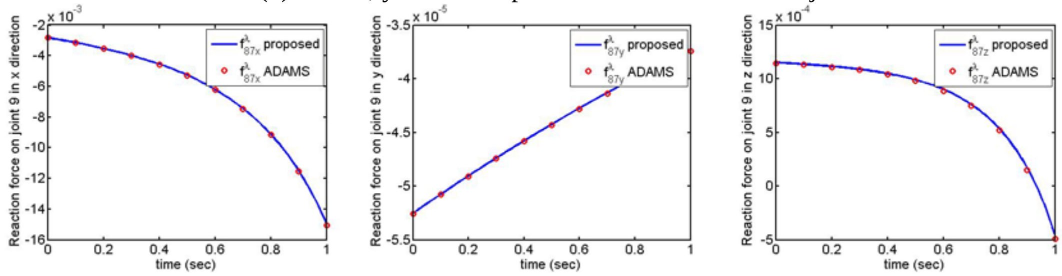
In order to validate the results, a CAD model is developed in ADAMS software environment. The outputs of the ADAMS model are also shown in the same figures that indicate good agreement of the analytical results and results of ADAMS model.



(a) Driving force at each prismatic joint



(b) The x, y and z components of reaction force at joint 8



(c) The x, y and z components of reaction force at joint 9

Figure 4: Comparison of analytical and ADAMS inverse dynamic results of the 3-RPS parallel manipulator

5 Conclusions

In this paper, the dynamic analysis of a three-leg structure, namely, 3-RPS parallel manipulator is proposed. Euler parameters are utilized to define orientation of its platform due to spherical joints motion. The advantage of the formulation is its ability to take away the singularity of the manipulator, since it is well-known that there is no inherent geometrical singularity in their equations. Then, equations of motion are developed using the concept of DeNOC matrices. The results are compared with those of the manipulator simulation in ADAMS environment. The outcome demonstrates good agreement between the results obtained from the algorithm and those are found from ADAMS software.

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