Vibration based Fatigue Damage Assessment of Cantilever Beams

N. Harish Chandra , A.S. Sekhar

Abstract

This paper explores to relate total fatigue damage and frequency of vibration resulting from excitation of a cantilever beam. From recent studies on damage mechanics, it is found that the magnitude of total fatigue damage of a component not only depends on the induced dynamic strain but also on the severity of vibrations. The Hybrid I-kaz method provides a two dimensional graphical representation of the measured strain and vibration signal. Hybrid I-kaz parameter (Z_h) provides a measure for the degree of data scattering and thus incorporating strain-vibration effects into the relation between frequency and total fatigue damage. The comparative study is implemented using strain-life approach, and Hybrid Integrated kurtosis based algorithm for Z notch filter technique (Hybrid I-kaz). Two sets of the experiments are conducted, considering the frequencies near first mode and second mode of the beam. The pattern obtained by frequency variation with the Hybrid kurtosis parameter is useful in finding hybrid kurtosis parameter at any given frequency of vibration. It is found that the total fatigue damage is varying linearly with the hybrid kurtosis parameter. This relation is proved to be useful in predicting fatigue damage of a beam at any frequency of vibration.

Keywords: Vibrations, Hybrid I-Kaz Method, Total Fatigue Damage.

1 Introduction

Fatigue is a problem in design of many structures. It is obvious that fatigue damage can be different in two cases, where dynamic strains are identical but the induced vibrations are different. Benefits of using vibrational velocity measurements for the estimation of fatigue damage are that the measurements are easily performed, the transducers are portable and robust, it is not necessary to know the location of maximum dynamic stress, and the positioning of transducers is not critical to the accuracy of dynamic stress predictions. Limitations with the use of strain gauges to measure maximum dynamic stress levels as explained by Norton [1] are that (i) at higher frequencies and short wavelengths, the magnitude of dynamic stress decreases rapidly with distance from boundaries and discontinuities where dynamic stress is usually high, making the measurement of maximum dynamic stress using finite length strain gauges is very difficult and (ii) the location of maximum dynamic stress for installation of strain gauges is not known.

Recent progress in random fatigue, fracture, and damage analysis is summarized and reviewed by Yao [2]. A solution for the power spectral density data is derived by Dirlik [3] and is useful in this study for finding the fatigue life in frequency domain. George et al. [4, 5] have developed a new vibration-based fatigue testing methodology. Abdullah [6] observed that the total fatigue damage is linearly

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proportional to the hybrid kurtosis number for a spring suspension system excited at different frequencies. More studies related to Hybrid kurtosis parameter are published by Nuawi [7] and Ismail [8]. Fatigue life estimation in time and frequency domains for plates is discussed by Ariduru [9]. Standard practices for cyclic counting in fatigue analysis by ASTM are useful in life estimation [10].

The objective of this study is to find a relation between total fatigue damage (D) and excitation frequency (f) in terms of hybrid kurtosis parameter (Z_h). The focus is to incorporate, both effects of strain and vibration severity into a method to find the fatigue damage.

2 System and Methodology Description

2.1 System description

A cantilever aluminum beam of size 690 x 75 x 6 mm is used for the experiments. The strain gages are biased towards the clamped end to improve strain sensitivity. A vibration shaker is mounted below the cantilever beam near the clamped end. This shaker is installed along with a function generator to excite the beam at required frequencies. An accelerometer is attached at the free end of the cantilever beam. The schematic of the experimental setup is presented in Fig. (1).

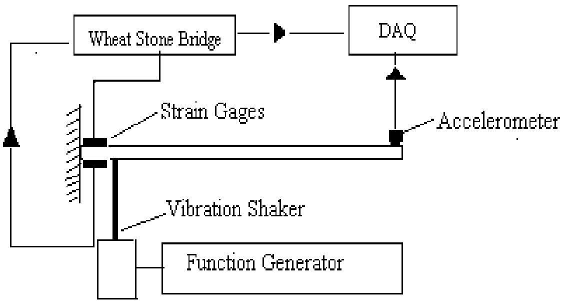


Figure 1: Schematic diagram of Experimental Setup

2.2 Methodology

To evaluate the fatigue life of the beam by vibration based method the following procedure is implemented. The theoretical modal frequencies for this beam are computed based on the known solution [11] and are compared with experimental modal frequencies obtained from the Fast Fourier Transform of an impulse response.

To measure the degree of data scattering, the Hybrid I-kaz parameter [6] is estimated. Hybrid Kurtosis or I-kaz Parameter (Z_h) provides a two dimensional graphical representation of the measured strain and vibration signal. Primary objective is to find a relation between Z_h and the frequency (f) of excitation. The correlation study is performed by considering the total fatigue damage and Hybrid I-kaz coefficients, Z_h for each signal of the different frequencies. Kurtosis control techniques increase the number of large vibration peaks in the random vibration test while maintaining the same RMS values and the same power spectral density (PSD). These peak vibrations are primarily responsible for the damage that a product experiences in the field. Some of the kurtosis control techniques are capable of getting the kurtosis to affect the data at high frequencies, but fail to be effective at lower frequencies. Hybrid Kurtosis is proved to be effective in affecting data at lower frequencies. The Hybrid I-kaz Coefficient, Z_h is given in Eq. (1).

$$Z_h = \frac{1}{n}\left(\sqrt{K_\varepsilon S_\varepsilon{}^4 + K_v S_\upsilon{}^4}\right) \tag{1}$$

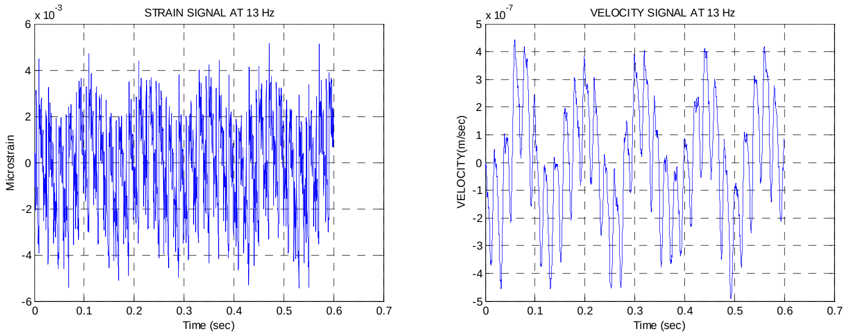
Where n is the number of data, K_ε and K_υ are the kurtosis of the strain and vibration signals, respectively and S_ε and S_υ are the standard deviation of the strain and vibration signals, respectively. The standard deviation (S) and Kurtosis of any variable X for n data points are mathematically defined by Eq. (2) and Eq. (3) respectively.

$$S = \sqrt{\left\{\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2\right\}} \tag{2}$$

$$K = \frac{1}{n(X_{rms})^4}\sum_{i=1}^{n}(X_i - \bar{X})^4 \tag{3}$$

Kurtosis, which is the signal 4th statistical moment, is a global signal statistic which is highly sensitive to the spikiness of the data. Higher kurtosis values indicate the presence of more extreme values than should be found in a Gaussian distribution. In industry, kurtosis is implemented to detect highly sensitive faults as it is found to more sensitive to high amplitude events.

Finite element analysis (FEA) performed in this work using ANSYS 12.1 software determines the highest static strain position.



(i) (ii)

Figure 2: At First Mode (i) Dynamic Strain and (ii) Velocity of vibration

The amplified analogue signals from the accelerometer and the strain gage are acquired by means of 16 bit acquisition board. A "Dewesoft" program, especially developed for this purpose activates the acquisition board and processes the signals. In every measurement, both the strain gage and the vibration signal are acquired at a time for a particular frequency. The strain and vibration signals at the 1st natural frequency of the beam (13Hz) are presented in Fig. (2). Frequency dependence of hybrid kurtosis coefficient is evaluated by regression analysis.

2.2.1 Total fatigue damage estimation (D)

In frequency domain analysis, power spectral density function estimates of normal stress are obtained from the acquired strain data sampled at 1000 Hz. The moments of the power spectral density estimates are used to find the probability density function estimate from Dirlik's empirical expression [3]. A MATLAB program based on Dirlik's approach is used for this purpose. After the total number of cycles in frequency domain approaches is found, Palmgren-Miner rule is used to estimate the damage at different frequencies. Total Fatigue Damage is obtained as the steps of the process given in Fig. (3).

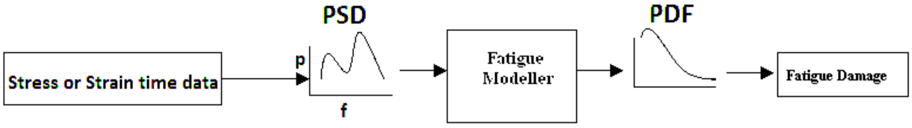


Figure 3: General procedure for frequency domain fatigue life calculation [9]

In signal analysis, the fatigue damage assessment and the Hybrid I-kaz method are evaluated for each collected signal. Finally, the correlation analysis is performed in term of the total fatigue damage. The methodology used in the present study is given in Fig. (4).

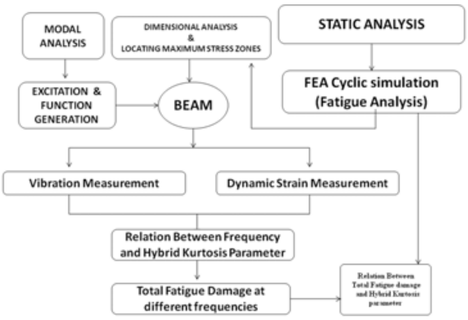


Figure 4: Methodology

3 Results and Discussion

3.1 Modal and statistical analysis

The five lowest natural frequencies of the cantilever beam system are evaluated using modal analysis as explained in section 2.2. These results are compared with those of the theoretical analysis in Table 1. The time series of 3000 data points is used to obtain scatter plots between dynamic strain and velocity at different frequencies. It is observe that a scatter plot between strain and velocity signals is an ellipse at first mode. For frequencies below and above the first mode the ellipse appeared more distorted. This distortion or scattering of data is quantified with the aid of Hybrid kurtosis parameter. The statistical analysis results are summarized in Table 2 and Table 3.

Table 1: Modal Analysis

Sl.No.	*Natural frequencies (Hz) (Theoretical)*	*Natural frequencies (Hz) (Experimental)*
1.	*11.2*	*13.0*
2.	*65.0*	*65.2*
3.	*182.6*	*190.0*
4.	*363.6*	*359.0*
5.	*589.4*	*585.0*

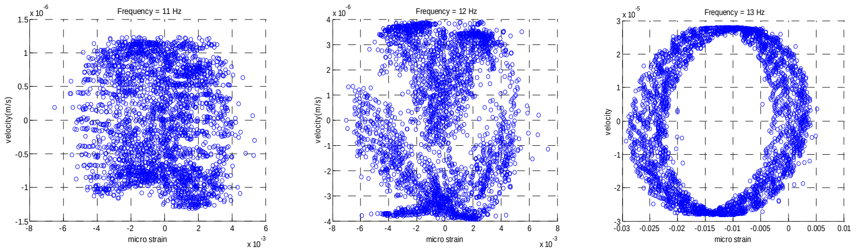
Table 2: Statistical Parameters and Hybrid kurtosis near first mode

Frequency (Hz) (f)	*Kurtosis Strain* (K_s)	*Kurtosis Velocity* (K_v)	*Std. Deviation Strain* (S_s)	*Std. Deviation Velocity* (S_v)	*Hybrid kurtosis number* $(Z_h)x\ 10^{-9}$
7	*2.3070*	*2.2642*	*0.001977*	*1.62E-07*	*1.98*
8	*2.1339*	*2.3252*	*0.002100*	*2.12E-07*	*2.07*
9	*2.2695*	*2.0109*	*0.002100*	*3.20E-07*	*2.23*
10	*2.3185*	*1.6666*	*0.002000*	*5.40E-07*	*2.13*
11	*2.2544*	*1.6413*	*0.002200*	*7.39E-07*	*2.34*
12	*2.3635*	*1.5256*	*0.002600*	*2.61E-06*	*3.59*
12.5	*2.3469*	*1.5071*	*0.003000*	*3.55E-06*	*4.45*
12.8	*2.0059*	*1.4876*	*0.004400*	*7.68E-06*	*9.19*
13	*1.6335*	*1.5113*	*0.009400*	*1.96E-05*	***37.8***

Table 3: Statistical Parameters and Hybrid kurtosis near second mode

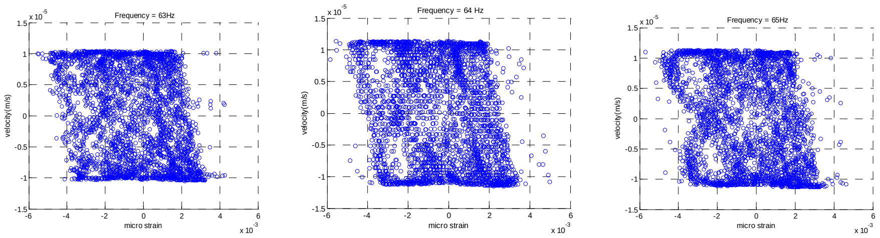
Frequency (Hz) (f)	*Kurtosis Strain* (K_s)	*Kurtosis Velocity* (K_v)	*Std. Deviation Strain* (S_s)	*Std. Deviation Velocity* (S_v)	*Hybrid kurtosis number* $(Z_h)x\ 10^{-9}$
61	*2.1181*	*1.5007*	*0.002*	*6.81E-06*	*2.02*
62	*2.0484*	*1.4987*	*0.002*	*7.23E-06*	*1.96*
63	*2.0928*	*1.503*	*0.002*	*8.51E-06*	*2.01*
64	*2.3376*	*1.4978*	*0.002*	*9.07E-06*	*2.10*
65	*2.1238*	*1.5005*	*0.0021*	*1.05E-05*	*2.11*

In Fig. (5) and Fig. (6) the significance of the hybrid kurtosis parameter is illuminated. The hybrid kurtosis parameter is found increasing with the frequency. At the first natural frequency of the beam, the scatter plot appeared to be an ellipse and the distortion is clearly visible with change in frequency. At second natural frequency, the scatter plot took a dumbbell shape as shown in Fig. (6).



(i) (ii) (iii)

Figure 5: Scatter plot for velocity and strain, at frequency, (i) 11Hz, (ii) 12 Hz (iii) 13 Hz



(i) (ii) (iii)

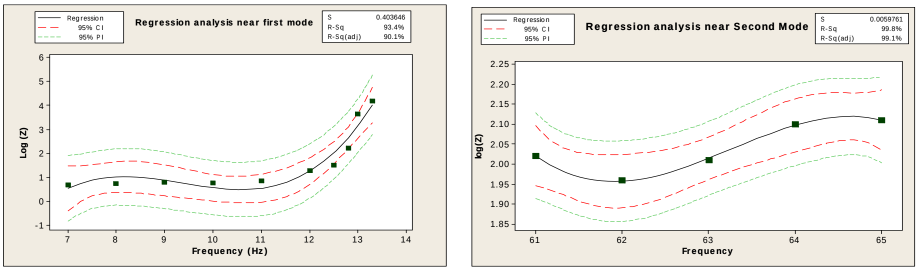
Figure 6: Scatter plot for velocity and strain, at frequency (i) 63 Hz, (ii) 64 Hz (iii) 65 Hz

3.2 Relation between frequency and hybrid kurtosis coefficient

The measured strain is used as the input information for the cyclic-based FEA calculation. The regression Eq. (4) and Eq. (5) relate Hybrid kurtosis coefficient (Z_h) and frequencies (f) near first mode and second mode of vibration respectively. The regression plots are shown in Fig. (7). The analysis is restricted at frequencies near first and second modes since the fatigue damage is assumed to be high at resonance.

$$Log\ (Z_h) = -79.13+19.57(f)-2.120(f^2) +0.07524(f)^3 \qquad (4)$$

$$Log\ (Z_{h)} = 864.3–41(f) +0.6484(f)^2-0.003416(f)^3 \qquad (5)$$

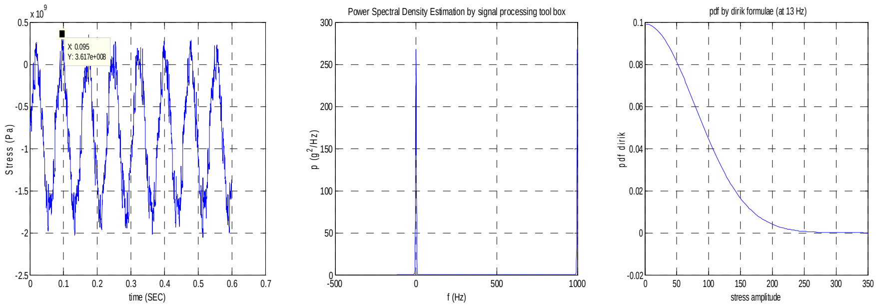


(i) (ii)

Figure 7: Cubic regression analysis of Hybrid Kurtosis and Frequency (Hz) near (i) first mode (13 Hz) and (ii) second mode (65.2 Hz) of the beam

3.3 Total fatigue damage

The fatigue analysis of the test specimen can be carried out in time and frequency domains. In frequency domain analysis, power spectral density function (PDF) of normal stress or strain is obtained from the acquired strain data sampled at a frequency. The moments of the power spectral density (PSD) estimates are used to find the probability density function estimate from Dirlik's empirical expression [3]. In Fig. (8) a stress signal at 13 Hz frequency , the PSD estimate sampled at 1000Hz and the PDF by Dirlik's solution are shown. After the total number of cycles in frequency domain approach is found, Palmgren- Miner rule is used to estimate the fatigue life. Stress signal obtained from strain time data is used as input to the Matlab program.



(i) (ii) (iii)

Figure 8: (i) stress signal obtained from strain time data at 13 Hz frequency, (ii) the PSD estimate sampled at 1000Hz and (iii) the PDF by Dirlik's solution

Total damage is calculated by dividing the number of cycles found in the frequency domain for each stresses (S) to the number of cycles (N) found from the Eq. (7) for Aluminum.

$$\text{Log}(S)=4.23-0.4\times\log(N) \tag{6}$$

$$N = 10^{\frac{4.23-\log(s)}{0.48}} \tag{7}$$

For every frequency the number of cycles is found separately. According to the Palmgren-Miner rule, the damage fraction at any stress level S_i is the sum of ratio of number of cycles of operation (n_i) to the total number of cycles (N_j) that produces failure at that stress level over k blocks, that is

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \tag{8}$$

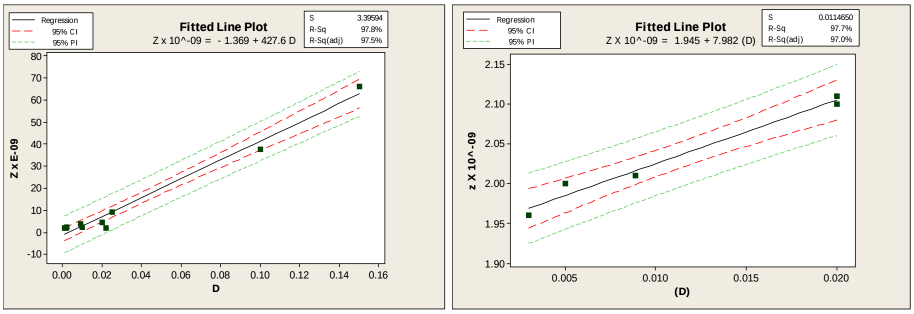
And the event of failure can be defined as

$$D \geq 1.0 \tag{9}$$

3.4 Relation between hybrid kurtosis and total fatigue damage

The relation between Z_h and D is established by regression analysis. The linear regression appears to provide a good fit to the data shown in Table 4 and 5; Eq. (10) and Eq. (11) represent the regression equations for frequencies near first and second modes. The Coefficient of determination (R^2) value showed in Fig. (9), indicates that coefficient (Z_h) accounts for 97% of the damage (D). This R^2 value provides a measure of how well future outcomes are likely to be predicted by the model. A visual inspection of the plot shown in Fig. (9), reveals that the data is randomly spread about the regression line, implying no systematic lack-of-fit.

The equations (10) and (11) can be generalized as per Eq. (12), the α and β coefficients will be different for different materials and components. Thus Total fatigue damage (D) can be obtained by using equations (4) or (5) and (12). The Total fatigue damage (D) estimated using equation (12) for different frequencies are compared with those obtained using FEM by ANSYS and are given in Table 6.



(i) (ii)

Figure 9: Relation between Hybrid Kurtosis and Total Fatigue Damage near (i) First Mode (ii) Second Mode

Table 4: Z_h and D near First Mode

Frequency(f) (Hz)	*Hybrid kurtosis number (Z_h)X 10^{-9}*	*Total fatigue Damage (D)*
7	*1.98*	*0.0010*
8	*2.07*	*0.0019*
9	*2.23*	*0.0020*
10	*2.13*	*0.0220*
11	*2.34*	*0.0100*
12	*3.59*	*0.0090*
12.5	*4.45*	*0.0200*
12.8	*9.19*	*0.0250*
13	***37.8***	*0.1000*
13.3	*66.3*	*0.1500*

Table 5: Z_h and D near Second Mode

Frequency (f) (Hz)	*Hybrid kurtosis number (Z_h)X 10^{-9}*	*Total fatigue Damage (D)*
61	*2.00*	*0.0050*
62	*1.96*	*0.0030*
63	*2.01*	*0.0089*
63	*2.10*	*0.0200*
65	*2.11*	*0.0200*

$$Z_h \text{ x } 10^{-9} = -1.369 + 427.6 \text{ (D)} \quad \text{for frequencies near first mode} \qquad (10)$$

$$Z_h \text{ x } 10^{-9} = 1.945+7.9 \text{ (D)} \quad \text{for frequencies near second mode} \qquad (11)$$

$$Z_h \text{ x } 10^{-9} = \alpha + \beta \text{ (D)} \quad \text{at any frequency} \qquad (12)$$

Table 6: Total Fatigue Damage (D) at different frequencies

Sl. No	*Frequency (f) in Hz*	*Hybrid kurtosis Parameter (Z_h)x 10^{-9}*	*Total Fatigue Damage (D)*	
			Using Equation (12)	*Using Ansys Fatigue Tool*
1	*20*	*7.23*	*0.0201*	*0.0211*
2	*30*	*5.61*	*0.0153*	*0.0141*
3	*40*	*4.59*	*0.0139*	*0.0160*

4 Conclusions

Focus of this study is to relate total fatigue damage and frequency of vibration by Hybrid kurtosis parameter of dynamic strain and velocity signals. The results show that the total fatigue damage varies linearly with hybrid kurtosis coefficient. The above derived relation can be used to find fatigue damage of a cantilever beam at a particular frequency. A general equation for fatigue damage is derived and verified.

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