Dynamically Stable Gait Planning of a Three-Legged Vertical Surface Climbing Robot

Tarun Kumar Hazra and Nirmal Baran Hui

Abstract

In this paper an attempt has been made to design a three legged vertical robot. Each leg of the robot is considered to have two links connected each other and with the trunk by revolute joint. The legs are symmetrically distributed over the trunk body. Movement of the Center of Gravity (CG) of the robot has been ensured considering one leg is moving at a time and other two are providing necessary support to the robot body. Motion equations of the robot have been derived using the concept of Newtonian mechanics. Thereafter, stability of the robot is analysed. Computer simulations have been carried out to test the navigation capability of the robot on a vertical surface. Performances are found to be satisfactory and computational complexity of the developed algorithms is found to be low.

Keywords: Tripod Robot, Kinematics, Gait Planning, Dynamics, Stability.

1 Introduction

Quite a large number of researchers devoted their time in the development of legged robots. There are many advantages of using legged robots over the wheeled robots and some disadvantages too. The main disadvantage is that a legged robot needs to plan both its path as well as gait (the sequence of leg movement) simultaneously during locomotion. However, it is extremely difficult job and complexity increases as the number of legs increases. As a result, stable gait generation of a hexapod robot is more critical than a quadruped robot. On the other hand, hexapod robot is more statically as well as dynamically stable than the quadruped or the biped one. It is because of the fact that for maintaining stability of a multi-legged robot, its projected center of gravity (CG) should lie within its support region, which is a convex hull passing through its supporting feet. As the number of legs reduces, number of supporting feet reduces and the convex hull becomes smaller. Therefore, it is a fertile area of research and many unsolved research problems still exist. Most important job to design a legged robot is to solve kinematics, dynamics, gait planning and its stability during locomotion. Denavit and Hartenberg [1] had proposed an approach to solve the kinematics of different kind of robots. In order to solve the dynamics of the robot, some researchers follow Lagrange Euler mechanics [2], and other follow Newtonian mechanics [3]. There are other researchers who have worked on the development of the robot's motion planning [4]. Stability is another important criterion for legged robot. There are two types of stability: static and dynamic [5, 6].

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Different aspects of stability of the robot have been explored by various researchers [7-8].

Going through the literature, it has been observed that the research on the robot having odd number of legs is limited. It may be due to the fact that gait planning of robots having odd number of legs is extremely difficult. Bretl et al. [4] tried to model a three-legged robot climbing natural terrain. On the other hand, it is necessary to analyse kinematics and dynamics of any robot before assessing its stability or controlling it. In this context, it is important to mention that the three-limbed robot is statically stable (since three point-based supports are statically stable). During locomotion at least one leg must be in swing phase (i.e., ground reaction forces in that leg would be zero) and it results in instability. This problem becomes highly complex, if it is planned to move in the uneven surface.

Understanding the gap on the development of the three-limbed robot, an attempt is made in this work to design a tripod robot that will navigate in plain vertical surface. During this study, main focus is made on the following three issues: (a) kinematics (forward and inverse) analysis, (b) gait design and trajectory planning, and (c) dynamics and stability analysis of the tripod robot. Rest of the paper is structured in the following manner. In Section 2, both forward as well as the inverse kinematics of the said robot has been discussed and foot trajectory planning of the robot has been explained. Formulation of the dynamics model along with the stability analysis has been presented in Section 3. Results are presented and discussed in Section 4. Finally, some concluding remarks have been made and scope for future work has been indicated in Section 5.

2 Kinematic Model and Trajectory Generations

In this paper, an attempt has been made to develop a suitable model of a planar three legged robot as shown in Fig. 1. A possible kinematic posture of the robot model is shown in Fig. 2. The robot consists of three legs and a trunk of triangular cross section with each side is equal to '*a'*. Each leg has two links connected each other and with the trunk by two rotary joints. It is also important to mention that each joint will be controlled separately using differential drive DC servo motors. The Denavit-Hartenberg (D-H) notations [1] have been followed in kinematic modeling of each leg. Table 1 shows four D-H parameters of a leg (say, i), namely link length (a_{j-1}), link twist (α_{j-1}), joint offset (d_j) and joint angle (θ_j) (here 'j' denotes a joint connecting two successive links) by following the concept described in Craig [9]. It is important to mention that for simplicity, link lengths of all the legs are made same. Therefore, the first link of a leg, which is connected to the trunk, is denoted by L_1 and second link is represented by L_2. From the above relationship, differences between the coordinates of the foot tip point (x_{end}^i, y_{end}^i, z_{end}^i) and CG (x_c, y_c, z_c) of i-th leg can be determined for the supplied joint variables (θ_2^i and θ_3^i) as follows.

$$\left.\begin{aligned} p_x^{\mathrm{i}} &= (x_{end}{}^{i} - x_c) = a/2\sqrt{3}c_1{}^{i} + L_1 c_{12}{}^{i} + L_2 c_{123}{}^{i}, \\ p_y^{\mathrm{i}} &= (y_{end}{}^{i} - y_c) = a/2\sqrt{3}\mathrm{s}_1{}^{\mathrm{i}} + \mathrm{L}_1\mathrm{s}_{12}{}^{\mathrm{i}} + \mathrm{L}_2\mathrm{s}_{123}{}^{i}, \\ p_z^{\mathrm{i}} &= (z_{end}{}^{i} - z_c) = -h \end{aligned}\right\}, \tag{1}$$

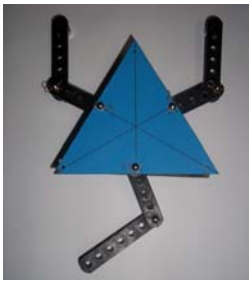


Figure 1: A planar three-legged robot.

Table 1: D-H parameter table for leg-i.

Joint No. (j)	α_{j-1}	a_{j-1}	d_j	θ_j
CG	0	0	0	$\theta_1^{\ i}$
1	0	$a/2\sqrt{3}$	0	$\theta_2^{\ i}$
2	0	L_1	0	$\theta_3^{\ i}$
Tip point	0	L_2	-h	0

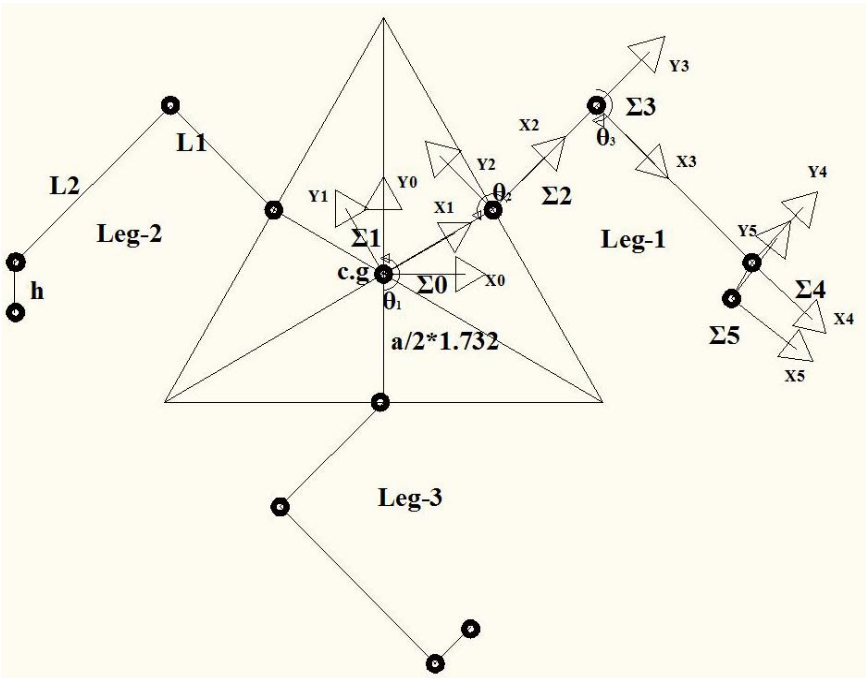


Figure 2: A 2D schematic sketch showing the frames assigned to the first leg of the robot.

Solving Eq. (1) algebraically, the joint angles $\theta_2^{\ i}$ and $\theta_3^{\ i}$ can be calculated. There will be two solutions for each posture of the robot and two values of $\theta_3^{\ i}$ can be determined as

$$\theta_{31}^{\ i} = \text{atan2}\left(s_3^{\ i}, c_3^{\ i}\right), \text{ and } \theta_{32}^{\ i} = \text{atan2}\left(-s_3^{\ i}, c_3^{\ i}\right) \tag{2}$$

where $c_3^{\ i}$ can be found out from the following equation

$$c_3^{\ i} = \frac{\left[\left\{p_x^{\ i} - \frac{a}{2\sqrt{3}} c_1^{\ i}\right\}^2 + \left\{p_y^{\ i} - \frac{a}{2\sqrt{3}} s_1^{\ i}\right\}^2 - \left\{L_1^{\ 2} + L_2^{\ 2}\right\}\right]}{2L_1L_2}$$

It is important to note that the value of $c_3^{\ i}$ must lie between -1 and 1 and knowing the values of $\theta_3^{\ i}$, two values of $\theta_2^{\ i}$ can be obtained as

$$\begin{aligned}
\theta_{21}^{\,i} &= \operatorname{atan2}\left[\left\{p_y^{\,i} - \frac{a}{2\sqrt{3}} s_1^{\,i}\right\}, \left\{p_x^{\,i} - \frac{a}{2\sqrt{3}} c_1^{\,i}\right\}\right] - \varphi_{31}^{\,i} \\
\theta_{22}^{\,i} &= \operatorname{atan2}\left[\left\{p_y^{\,i} - \frac{a}{2\sqrt{3}} s_1^{\,i}\right\}, \left\{p_x^{\,i} - \frac{a}{2\sqrt{3}} c_1^{\,i}\right\}\right] - \varphi_{32}^{\,i} \\
\text{where } \varphi_{31}^{\,i} &= \operatorname{atan2}\left(L_1 s_1^{\,i} + L_2 \sin\left(\theta_1^{\,i} + \theta_{31}^{\,i}\right), L_1 c_1^{\,i} + L_2 \cos\left(\theta_1^{\,i} + \theta_{31}^{\,i}\right)\right) \\
\varphi_{32}^{\,i} &= \operatorname{atan2}\left(L_1 s_1^{\,i} + L_2 \sin\left(\theta_1^{\,i} + \theta_{32}^{\,i}\right), L_1 c_1^{\,i} + L_2 \cos\left(\theta_1^{\,i} + \theta_{32}^{\,i}\right)\right)
\end{aligned} \tag{3}$$

2.1. Gait Planning Strategy

During locomotion, a one-step movement is normally followed by human being and Bretl et al. [4] have mentioned that one step movement can also be used for planning gaits of three-legged robots. In the present work the gaits of the robot have also been planned in the similar manner. It has been assumed that during movement at a time only one leg will be in swing phase and other two will be in support phase. At first say leg-2 and leg-3 are in support phase and leg-1 is in the swing phase moving along a specified path. When leg-1 will reach to its goal configuration, leg-1 and leg-3 will switch to support phase, and leg-2 will be in swinging phase. Thereafter, leg-3 will be in swing phase and the other two will be in support phase. In this way the tip of legs of the tripod reach to the new position with different configurations and completes one locomotion cycle.

Let us say, initial position of the geometric center of the robot is (x_c, y_c, z_c). The CG of the robot is moving in a straight line path along 'Y' axis with a constant speed. The time for a full locomotion cycle is considered to be equal to Δt. After Δt time interval the new position of the CG is (x_c, $y_c+\Delta y$, z_c). The above movement is achieved in three stages. For each stage, coordinates of the CG of the robot, foot tip point of three legs at different instant of time have been presented in Table 2.

Table 2: Positions of different parts of the robot at four instant of time of a locomotion cycle.

Time ($t = k\Delta t$)	CG	Foot tip point		
		1st leg	2nd leg	3rd leg
k=0	x_c, y_c, z_c	x_1, y_1, z_1	x_2, y_2, z_2	x_3, y_3, z_3
k=1/3	$x_c, (y_c + \Delta y/3), z_c$	$x_1, (y_1 + \Delta y), z_1$	x_2, y_2, z_2	x_3, y_3, z_3
k=2/3	$x_c, (y_c + 2\Delta y/3), z_c$	$x_1, (y_1 + \Delta y), z_1$	$x_2, (y_2 + \Delta y), z_2$	x_3, y_3, z_3
k=1	$x_c, (y_c + \Delta y), z_c$	$x_1, (y_1 + \Delta y), z_1$	$x_2, (y_2 + \Delta y), z_2$	$x_3, (y_3 + \Delta y), z_3$

In this paper, the joint trajectory is interpolated as a linear function with parabolic blend at the beginning and end of the trajectory as explained in Craig [9].

3 Dynamics of the Robot

Dynamic equations for both the protraction and retraction phases have been derived separately using Newton-Euler based formulation. Torque expressions [10] at the Joints 1 and 2 for the i-th leg are given as follows.

Retraction Phase:

$$\tau^i_{1R} = \begin{bmatrix} \frac{1}{3}m^i_1(L_1)^2 + m^i_2(L_1)^2 \\ +\frac{1}{3}m^i_2(L_2)^2 + m^i_2 L_1 L_2 c^i_2 \end{bmatrix} \ddot{\theta}^i_1 + \begin{bmatrix} \frac{1}{3}m^i_2(L_2)^2 \\ +\frac{1}{2}m^i_2 L_1 L_2 c^i_2 \end{bmatrix} \ddot{\theta}^i_2 + \begin{bmatrix} L_2\left(F^i_{g2X}s^i_3 + F^i_{g2Y}c^i_3\right) + \\ L_1\left(F^i_{g2X}s^i_{23} + F^i_{g2Y}c^i_{23}\right) \end{bmatrix} \quad (4)$$

$$-\frac{1}{2}m^i_2 L_1 L_2 s^i_2 \dot{\theta}^i_2\left(2\dot{\theta}^i_1 + \dot{\theta}^i_2\right) + \frac{1}{2}m^i_2 L_2 g_a c^i_{12} + \frac{1}{2}m^i_1 L_1 G c^i_1 + m^i_2 L_1 g_a c^i_1$$

$$\tau^i_{2R} = \begin{bmatrix} \frac{1}{3}m^i_2(L_2)^2 \\ +\frac{1}{2}m^i_2 L_1 L_2 c^i_2 \end{bmatrix} \ddot{\theta}^i_1 + \frac{1}{3}m^i_2(L_2)^2\ddot{\theta}^i_2 + \frac{1}{2}m^i_2 L_1 L_2 s^i_2\left(\dot{\theta}^i_1\right)^2 \quad (5)$$

$$+\frac{1}{2}m^i_2 L_2 g_a c^i_{12} + L_2\left(F^i_{g2X}s^i_3 + F^i_{g2Y}c^i_3\right)$$

Protraction Phase:

$$\tau^i_{1P} = \begin{bmatrix} \frac{1}{3}m^i_1 L_1^2 + m^i_2(L_1)^2 \\ +\frac{1}{3}m^i_2 L_2^2 \\ +m^i_2 L_1 L_2 c^i_2 \end{bmatrix} \ddot{\theta}^i_1 + \begin{bmatrix} \frac{1}{3}m^i_2 L_2^2 \\ +\frac{1}{2}m^i_2 L_1 L_2 c^i_2 \end{bmatrix} \ddot{\theta}^i_2 - \frac{1}{2}m^i_2 L_1 L_2 s^i_2 \dot{\theta}^i_2 \begin{pmatrix} 2\dot{\theta}^i_1 \\ +\dot{\theta}^i_2 \end{pmatrix} + \begin{bmatrix} \frac{1}{2}m^i_2 L_2 c^i_{12} \\ +\frac{1}{2}\begin{pmatrix} m^i_1 \\ +m^i_2 \end{pmatrix} L_1 c^i_1 \end{bmatrix} g_a \quad (6)$$

$$\tau^i_{2P} = \begin{bmatrix} \frac{1}{3}m^i_2(L_2)^2 \\ +\frac{1}{2}m^i_2 L_1 L_2 c^i_2 \end{bmatrix} \ddot{\theta}^i_1 + \left[\frac{1}{3}m^i_2(L_2)^2\right]\ddot{\theta}^i_2 + \left[\frac{1}{2}m^i_2 L_1 L_2 s^i_2\left(\dot{\theta}^i_1\right)^2\right] + \frac{1}{2}m^i_2 L_2 g_a c^i_{12} \quad (7)$$

Here, $m_1^{\mathrm{i}}, m_2^{i}, m_b$ denote mass of links 1 and 2 of i-th leg and body mass of the robot, respectively. Acceleration due to gravity is represented by g; speed of the CG of the robot is denoted by $\ddot{y}_c$ at i-th leg and g_a = [0 $(g+a_y)$ 0]^T, where a_y is the acceleration of the CG along y-direction. It is important to mention that all the joint torque expressions have been derived with respect to the coordinated frame attached to the CG of the robot.

Value for reaction forces are obtained as follows,

$$F_{2g2} = \sqrt{\left\{\left(F_{2g2X}\right)^2 + \left(F_{2g2Y}\right)^2\right\}} \quad \text{and} \quad F_{3g2} = \sqrt{\left\{\left(F_{3g2X}\right)^2 + \left(F_{3g2Y}\right)^2\right\}} \quad (8)$$

where,

$$F_{2g2Y} = \frac{\sin\left(\theta_{21}+\theta_{22}\right)}{L_{21}\sin\theta_{21}}\left[\tau'_{21}-\tau_{21}-\tau_{22}-m_{22}g\left\{L_{21}\cos\theta_{21}+\frac{L_{22}}{2}\cos\left(\theta_{21}+\theta_{22}\right)\right\}-m_{21}g\left\{\frac{L_{21}}{2}\cos\theta_{21}\right\}\right],$$

$$F_{3g2Y} = \left(m_{11}g + m_{12}g + m_{21}g + m_{22}g + m_{31}g + m_{32}g + m_b g\right) - F_{2g2Y},$$

$$F_{2g2X} = \left[\tau'_{22}-\tau_{22}-m_{22}g\left\{\frac{L_{22}}{2}\cos\left(\theta_{21}+\theta_{22}\right)\right\}-F_{2g2Y}\left\{L_{22}\cos\left(\theta_{21}+\theta_{22}\right)\right\}\right],$$

$$F_{3g2X} = -\left[\tau'_{22}-\tau_{22}-m_{22}g\left\{\frac{L_{22}}{2}\cos\left(\theta_{21}+\theta_{22}\right)\right\}-F_{2g2Y}\left\{L_{22}\cos\left(\theta_{21}+\theta_{22}\right)\right\}\right]$$

where, $\tau'_{11}, \tau'_{12}, \tau'_{21}, \tau'_{22}, \tau'_{31}, \tau'_{32}$ are net torques at different joint on leg-1, leg-2, and leg-3, respectively.

It is important to mention that determination of ground reaction forces is an iterative process and it depends on the requirement of torque at the joints during movement of the robot. In this paper, calculated torque in the present step has been used to obtain the reaction forces in the future step, which are finally fed to determine the torques at joints in that step. Although the similar analytical equations are used to determine the future & present step torques at joints, however, difference occurs as the position, velocity and accelerations of joints change with time.

Stability is an important criterion for the mobile robot. Depending upon the number of legs and the nature of the terrain the stability would be either static or dynamic. In order to describe the dynamic stability, some terminologies are used. Out of which, Zero Moment Point (ZMP), Fictitious Zero Moment Point (FZMP), Foot Rotation Indicator (FRI), Center of Pressure (COP), GPCOG [6-8] are important ones. In this work, an attempt has been made to asses the stability based on ZMP concept. ZMP is defined as that point on the ground at which the net moment of the inertia force and the gravity force has no component along the horizontal axes [7]. ZMP expressions are formulated as follows.

$$y_{ZMP} = \left(\sum_{i=1}^{3} \begin{pmatrix} \tau^i{}_1 + \tau^i{}_2 + m^i{}_1 \left(\ddot{y}^i_{c1} + g \right) x^i{}_{c1} \\ + m^i{}_2 \left(\ddot{y}^i_{c2} + g \right) x^i{}_{c2} \\ + m_{i1} y_{ic1} \ddot{x}_{ic1} + m^i{}_2 y^i{}_{c2} \ddot{x}^i_{c2} \end{pmatrix} + m_b \left(\ddot{y}_b + g \right) x_b + m_b y_b \ddot{x}_b \right) \Big/ \left(\sum_{i=1}^{3} \left(m^i{}_1 \ddot{x}^i_{c1} + m^i{}_2 \ddot{x}^i_{c2} \right) + m_b \ddot{x}_b \right) \quad (9)$$

$$x_{ZMP} = \left(\sum_{i=1}^{3} \begin{pmatrix} \tau^i{}_1 + \tau^i{}_2 + m^i{}_1 \left(\ddot{y}^i_{c1} + g \right) x^i{}_{c1} \\ + m^i{}_2 \left(\ddot{y}^i_{c2} + g \right) x^i{}_{c2} \\ + m_{i1} y^i{}_{c1} \ddot{x}^i_{c1} + m^i{}_2 y^i{}_{c2} \ddot{x}^i_{c2} \end{pmatrix} + m_b \left(\ddot{y}_b + g \right) x_{ic1} + m_b y_b \ddot{x}_b \right) \Big/ \left(\sum_{i=1}^{3} \left(m^i{}_1 \ddot{x}^i_{c1} + m^i{}_2 \ddot{x}^i_{c2} \right) + m_b \ddot{x}_b \right)$$

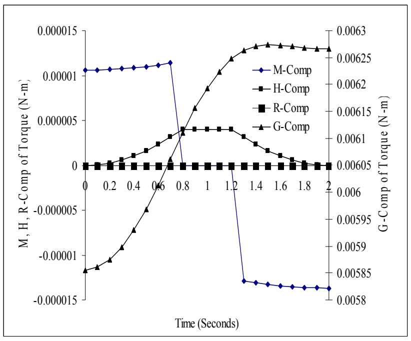
Following important expressions were used to determine the ZMP values.

$$ {}^2\dot{\mathrm{V}}^{\mathrm{i}}_{\mathrm{c2}} = \begin{bmatrix} \ddot{x}^i_{c2} \\ \ddot{y}^i_{c2} \\ \ddot{z}^i_{c2} \end{bmatrix} = \begin{bmatrix} \mathrm{L}_1 \ddot{\theta}^i_1 \mathrm{s}^i{}_2 - \mathrm{L}_1 (\dot{\theta}^i_1)^2 \mathrm{c}^i{}_2 + \mathrm{g_a} \mathrm{s}^i{}_{12} - \frac{\mathrm{L}_2}{2} \left(\dot{\theta}^i_1 + \dot{\theta}^i_2 \right)^2 \\ \mathrm{L}_1 \ddot{\theta}^i_1 \mathrm{c}^i{}_2 + \mathrm{L}_1 (\dot{\theta}^i_1)^2 \mathrm{s}^i{}_2 + \mathrm{g_a} \mathrm{c}^i{}_{12} + \frac{\mathrm{L}_2}{2} \left(\ddot{\theta}^i_1 + \ddot{\theta}^i_2 \right) \\ 0 \end{bmatrix}; \quad {}^1\dot{\mathrm{V}}^i_{\mathrm{c1}} = \begin{bmatrix} \ddot{x}^i_{c1} \\ \ddot{y}^i_{c1} \\ \ddot{z}^i_{c1} \end{bmatrix} = \begin{bmatrix} \mathrm{g_a} \mathrm{s}^i{}_1 - \frac{\mathrm{L}_1}{2} (\dot{\theta}^i_1)^2 \\ \mathrm{g_a} \mathrm{c}^i{}_1 + \frac{\mathrm{L}_1}{2} \ddot{\theta}^i_1 \\ 0 \end{bmatrix}; \quad \dot{\mathrm{V}}_{\mathrm{b}} = \begin{bmatrix} \ddot{x}_b \\ \ddot{y}_b \\ \ddot{z}_b \end{bmatrix} = \begin{bmatrix} 0 \\ \mathrm{a_y} \\ 0 \end{bmatrix}$$

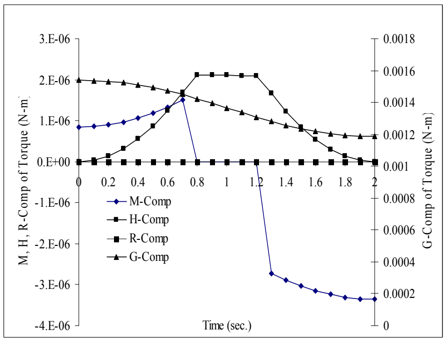
4 Simulation Results

Developed mathematical models have been tested through computer simulations. It has been carried out in a P-IV PC. In the present case, the leg stroke of the one step movement (Δy), body height (h), side length of the triangular-shaped cart (a) and time step are assumed to be equal to 0.03m, 0.05m, 0.12m and 6 seconds, respectively. During analysis, following data have been considered: L_1=0.04m, L_2=0.06m, m_1=0.002Kg, m_2=0.012Kg, m_b=0.075Kg, coordinates of CG of the robot at the starting point (0,0,0.05)m, foot tip points during starting for the first leg (0.09,0.05,0)m, for the second leg (-0.09,0.05,0)m and third leg at (0.01,-0.11,0)m.

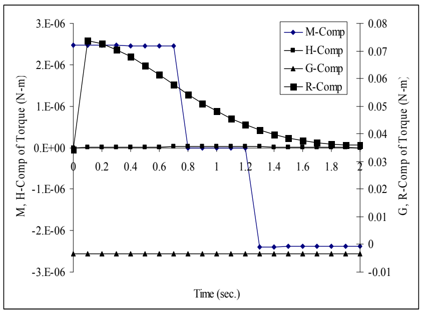
It is important to mention that forward kinematics always leads to a single pose matrix for any robot. However, several robot configurations (i.e., joint angle values) may result in the same foot tip point corresponding to a fixed location of the CG. In the present study, for each joint two solutions have been obtained corresponding to each joint variable and those combinations of solutions have been preferred which are falling within the reachable workspace of the robot. Utilizing the solutions obtained through inverse kinematics, three trajectories (one each in three different time zone: $0 \le t \le 2$; $2 \le t \le 4$ and $4 \le t \le 6$) for joint angle, speed and acceleration have been derived as mentioned in [9]. From Eqs. (6) & (7), it is clear that joint torques is comprised of four components: inertial component (M-Comp), centrifugal and/or Coriolis component (H-Comp), Gravity component (G-Comp) and reaction force component (R-Comp). Fig. 3 shows all those four components of each joint of the leg for the first phase of the locomotion cycle. Variation of total torque values of first and second joint of all the legs are also compared and presented in Figs. 4 and 5 for the second and third phases of the locomotion cycle, respectively.



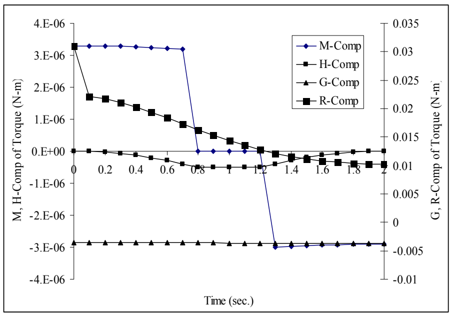
(a) Joint 1 of Leg-1,



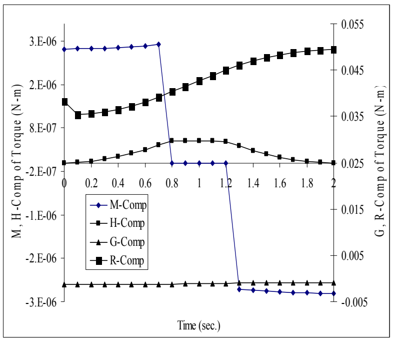
(b) Joint 2 of Leg-1,



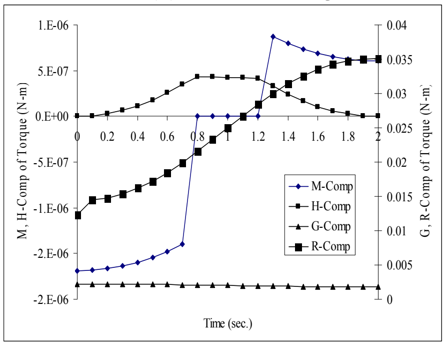
(c) Joint 1 of Leg-2,



(d) Joint 2 of Leg-2,

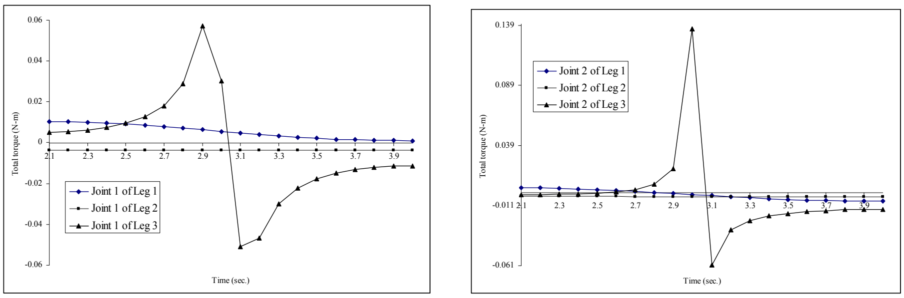


(e) Joint 1 of Leg-3,



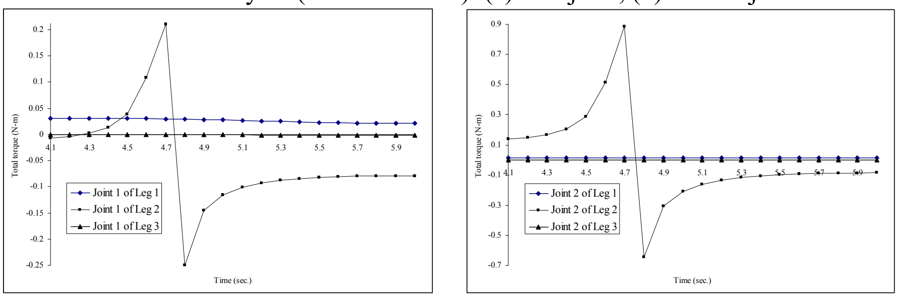
(f) Joint 2 of Leg-3,

Figure 3: Contribution of M-comp, H-comp, G-comp and R- comp of on the joint torques over the first phase of the locomotion cycle.



(a) First Joint (b) Second joint

Figure 4: Total torque (in N-m) distribution of each leg over the second phase of the locomotion cycle (time in second): (a) first joint, (b) second joint.



(a) First Joint (b) Second joint

Figure 5: Total torque (in N-m) distribution of each leg over the third phase of the locomotion cycle (time in second): (a) first joint, (b) second joint.

During this study, following common observations were made.

(i) During the first 1/4-th and last 1/4-th time period of every stages of locomotion cycle, inertial component of torque has been found to be approximately constant and in-between it is observed to be zero. It is because of the acceleration distributions considered in the study.

(ii) Reaction force component of torque has overpowered the others for retraction phase of every leg. It might be due to the fact that the motors alone are trying to supply the balance force required at the tip-point of the joint and there is no other device present to provide the same.

(iii) Whenever a leg is shifting its mode from support phase to swing phase, torque requirement on that leg has come out to be more than the others.

(iv) **For the first joint**: Contribution of gravity component in all the legs has been found to be more compared to the other two components. Gravity component has been observed to be varying in the positive side only for Leg 1. On the other hand it is varying both in positive as well as in negative side for the other two legs.

(v) **For the second joint:** Contribution of centrifugal and/or Coriolis component in all the legs is observed to be very low. It might be due to the fact that the first joint does not have any contribution on this component of torque.

(vi) Total torque requirement for the second joint of every leg has been found to be more in compared to the first joint during retraction phase. On the other hand, the thing comes out to be reverse during protraction phase. This is also due to the presence of reaction force.

5 Conclusions

Kinematics, dynamics and stability of a three-legged robot have been analysed in the present study. Both direct and inverse kinematics has been analysed, while the robot is following a straight line path on a vertical flat surface. Movement of the robot is ensured considering that at any instant of time only one leg can be in swing phase while the other two will provide necessary support. Joint torques has been computed continuously for the full locomotion cycle and compared between the legs. All the developed mathematical models have been tested through computer simulations. Computational complexity of the program developed for solving the mathematical expressions is found to be very low, making it suitable for on-line implementations. More torque requirement has been observed for the first joint of each leg and for every joint during support phase than in swing phase. For the first joint of each leg, torques varies between (-0.25N-m to 0.2N-m) and those for the second joint vary between (-0.7N-m to 0.9N-m). This is a very low torque requirement and low power servo motors will be sufficient to control them. A ZMP based stability analysis has been carried out during this study. It has been observed that the postures adopted during the vertically straight line movement of the robot are stable and feasible.

The present study can be extended in a number of ways, such as; dynamic stability analysis based on friction cone approach [4], optimization of joint torques or ground reaction forces of the robot while it is following a curvilinear path.

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