Task Space Trajectory Planning among Cooperating Robots through Mirror Motions

T A Dwarakanath, Gaurav Bhutani, Puneet Srivastava

Abstract

The co-operation among a set of robots is for the purpose of achieving a common goal. The intent of cooperation and objective of effective co-operation would set up a functional relationship among the co-operative robots. This paper presents a new trajectory planning scheme based on inter-relationship that has to exist in achieving a goal in cooperation. Apart from conceptualization of the scheme, the paper presents an analytical scheme and experimentally validates the concept using two in-parallel robots in cooperation. The development is based on the robotics in nuclear industry.

Keywords: Mirroring motions, Cooperative Robotics, Task Space Trajectory, Robot Trajectory Program.

1 Introduction

To plan and to program the participating robots individually towards a common goal at the task space is very complex [1]. The complexity is not only in programming the robots but also in operating and supervising them. This paper presents a new trajectory planning scheme based on the inter-relationship that has to exist in achieving a goal in cooperation. The paper deals with reliable and simple ways to divide and delegate the work to the participating robots. Identifying the functional relationship and setting the motion of the cooperating robots on mirror mode whenever possible would simplify the cooperative robot control, user interface and supervision. Many task space manipulations in cooperative robotics fall in the category of mirroring the motions at the task space of the cooperating robots. We have formulated a scheme which, intends to serve as a *Cooperating Robots Programming Interface* (CRPI). The idea behind the CRPI is to simplify the task space trajectory planning among cooperating robots using mirror motions. The development is based on the robotics in nuclear industry and the key feature of the work is to enhance the reliability by reducing the complexity.

Coordinated control of two and multi- robot arms has been studied by many investigators. Three kinds of basic approaches have been proposed for controlling two cooperative robots. The first approach adopts the model of master/slave [2], in which a robot is chosen as the master whose position is controlled, and the other robot is the slave whose position and acting force are controlled through a master. The second method is a centralized architecture which controls two robots by one controller. Only one controller controls multi robots working in cooperation. The

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third approach is a distributed controller [3], in which each robot is controlled separately. In support of the centralized controlling scheme, TARN [4] gave a formulation of cooperation of two robots; this formulation had the advantage of force control. In [5], load balancing and closed chain multiple arm control is discussed. Adaptive distributed cooperation controller for multiple manipulators are presented in [6], in which, only the position is controlled. For controlling the position and internal force of the whole system, a hybrid position/force method [5, 7] was proposed. YAO [8] gave a formulation for handling a fragile object by controlling position, internal and external force. The CRPI code can be implemented in all the three types of controls.

This paper deals with planning the delegation of task among co-operating robots such that the insight into the role of each participating robot is not eroded while performing a task. The cooperation among robots discussed in this paper is in position domain and is assessed based on a plan to introduce mirroring motions. The paper explains the concept of mirroring manipulation among cooperating robots and illustrates several applications that can be programmed under this framework. The context of the development is to automate several operations that take place inside the nuclear hot cell. The human intervention is very difficult; therefore the fixtures once mounted are not available for frequent changes. Safety and reliability are the high priority features. Cooperation of two robots that need to perform some common tasks in the hot cell is discussed. They are classified as pick and place, trace and cut, grip and hold, share payload and move. These functions of cooperation are applied to practical applications like welding, cutting, bending, gripping and screw turning actions. Planar parallel kinematic robots are used to perform many tasks in co-operation to demonstrate the concept. The concept can serve to automate several complex manipulations in the hot cell.

2 Cooperating Robots through Mirror Motions

The purpose of cooperation is to share the work in accomplishing the common goal. The categorization of tasks helps to plan the cooperation. For a cooperation, dividing the work and delegating the work to participating robot becomes utmost important. The simple option is to look for mirroring the efforts of participating robots in jointly handling the given task. In this paper, we address all categories of tasks under mirror motion scheme. We assume for all our planning purposes that the participating robots are identical in their kinematic structure and have the same dynamic capability. The experiments are conducted using planar parallel, 3 degree of freedom robots. Two translations in plane and a rotation about any axis normal to the plane constitute the 3 DOF of the robot. The robot can perform all types of motion in plane including coordinated translation along any line in a plane and a screw driver type rotation about any axis normal to the plane. The architecture of the manipulator is such that the major portion of the end effector is free of structural obstacles and can come face to face with its cooperating robot. Figure 1 shows the schematic of the two planar parallel robots. In a standard setup in the hot cell, a coordinate frame, $\mathbf{M}(\mathbf{X_M},\mathbf{Y_M},\mathbf{Z_M})$ is established at a point $\mathbf{M}$ and $[\hat{\boldsymbol{\imath}},\hat{\boldsymbol{\jmath}},\hat{\boldsymbol{k}}]^T$ are the unit vectors along $\mathbf{X_M}$, $\mathbf{Y_M}$ and $\mathbf{Z_M}$ respectively. The plane normal to the axis, $\mathbf{Y_M}$ and containing the point $\mathbf{M}$ is referred as the mirroring plane. Based on the mirroring plane and the workspace of the robot, the base frames, $\mathbf{O_1}(\mathbf{X_1}, \mathbf{Y1}, \mathbf{Z1})$ and $\mathbf{O_2}(\mathbf{X_2}, \mathbf{Y_2}, \mathbf{Z_2})$ of the participating robots are fixed. The base frames in mirror symmetry serve as reference co-ordinate frames for the participating robots. The standard setup is shown in figure 1.

In the following subsection, the mirror motion scheme is explained. The idea is also to develop a *Cooperating Robots Programming Interface* (CRPI). We describe the scheme by considering one task covering all types of cooperation that may take place in the hot cell between the two robots. Let us consider that the objective of the task is to generate a circular trace on the job as shown in figure 1. The condition is that the cooperation should not disturb the setup and the good feature of the cooperation should be such that the scope for human error is minimum.

2.1 Tracing about a mirror plane

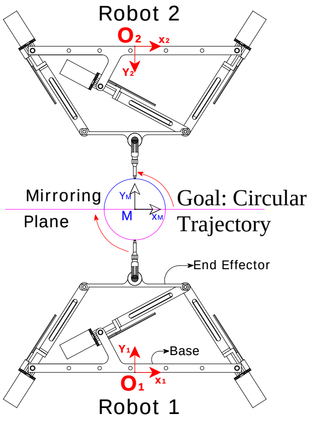

Figure 1: Two robots in mirror symmetry

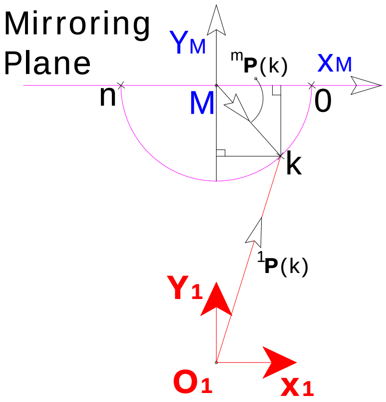

Figure 2: Task point description for Robot 1

A task is shared by both the participating robots and the task is such that the manipulation required from the participating robots is mirror symmetric about a mirror plane (**Z_M-X_M**). It is simple and many operations can be envisaged under this classification. The trajectory sharing about a mirror plane is explained by considering a circular trajectory, and the description will hold good for any trajectory that is mirror symmetric to the mirror plane. The circular task space trajectory is divided into two semi-circular trajectories about the mirroring plane. A point data, $\boldsymbol{k} \,\boldsymbol{\epsilon}\, [\boldsymbol{0}, \boldsymbol{n}]$ is the point on the circumference of the semi-circle (see figure 2) in the task space. The data delegation for the participating robots to trace a semi-circular trajectory is developed as follows.

$$ {}^{m}\boldsymbol{P}(\boldsymbol{k}) = {}^{m}\boldsymbol{P}(\boldsymbol{k})\hat{\boldsymbol{\imath}} + {}^{m}\boldsymbol{P}(\boldsymbol{k})\hat{\jmath} \quad (1) $$

$$ {}^{1}\boldsymbol{P}(\boldsymbol{k}) = \boldsymbol{O}_{1}\boldsymbol{M} + {}^{m}_{1}\boldsymbol{R}\,{}^{m}\boldsymbol{P}(\boldsymbol{k}) \quad (2) $$

Where, ${}^{m}\boldsymbol{P}(\boldsymbol{k})$ is the **k**th point with respect to the mirror frame, $\boldsymbol{O}_1\boldsymbol{M}$ is the known vector from **O_1** to **M.** ${}^{m}_{1}\boldsymbol{R}$ is the rotation of frame **M** with respect to **O_1.** ${}^{1}\boldsymbol{P}(\boldsymbol{k})$ is the **k**th point data in the task space for the participating robot 1, the parameters are as shown in the figure 2. An IKP solution on ${}^{1}\boldsymbol{P}(\boldsymbol{k})$, gives the joint data for robot 1. Similarly, all data points in the task space and the corresponding joint data for the participating robot 1 can be computed. The joint data and the corresponding instant of time is the control input for the robot motion. *The same data serves as the motion input file for robot 2 for the above setup and* is as given in equation 3.

$$[\boldsymbol{l_k}]_{R2} = [\,\mathrm{I}\,][\boldsymbol{l_k}]_{R1} \tag{3}$$

$[\boldsymbol{l_k}]_{Ri} = [l_{1k}, l_{2k}, l_{3k}]_{Ri}^{T}$ is the IKP solution of the kth point of the robot i and $[\,\mathrm{I}\,]$ is a 3×3 identity matrix. The robots 1 and 2 will start from an initial point, ${}^{\boldsymbol{m}}\boldsymbol{P}(\boldsymbol{0})$ and move in a mirror motion in opposite directions to end the trace at a point, ${}^{\boldsymbol{m}}\boldsymbol{P}(\boldsymbol{n})$ to accomplish a circular trace in the task space.

2.2 Sharing the trace by flipping the start and the end points

The aim is to complete the given trace by preserving the direction and flipping the start and the end points of the robots 1 and 2 as shown in figure 3. Many activities in the hot cell can be classified under this case. The simple way to achieve this is by mirroring $\mathbf{O_2}$ (rotating $\mathbf{O_2}$ by 180^0) about $\mathbf{Y_2}$. The same IKP solution set computed for robot 1 serve as the motion input file for the robot 2. However, this requires meddling with the fixture and may not be admissible. The preference is to achieve the motion by transformation of the input data file generated for robot 1. The transformation can be obtained by mirroring the robot 1 about $\mathbf{Y_1}$-$\mathbf{Z_1}$ plane. The input data file for robot 2 can be generated simply by interchanging columns of the data set of robot 1 as given in equation 4. Note that the inverse solution for a virtual link, shown as l_4 is also computed.

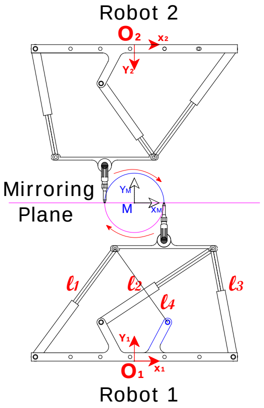

Figure 3: Cooperation in tracing a curve

$$[\boldsymbol{l_k}]_{R2} = [\,\mathrm{R}\,][\boldsymbol{l_k}]_{R1} \tag{4}$$

$$[\,\mathrm{R}\,] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix}_{R2} = [\,R\,] \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix}_{R1} = \begin{bmatrix} l_4 \\ l_3 \\ l_2 \\ l_1 \end{bmatrix}_{R1}$$

Where, $[\,\mathrm{R}\,]$ is a 4×4 reflection matrix. The robots 1 and 2 will start from an initial point, ${}^{\boldsymbol{m}}\boldsymbol{P}(\boldsymbol{0})$ and the mirror point of ${}^{\boldsymbol{m}}\boldsymbol{P}(\boldsymbol{n})$ respectively. They end the trace at a point, ${}^{\boldsymbol{m}}\boldsymbol{P}(\boldsymbol{n})$ and mirror point, ${}^{\boldsymbol{m}}\boldsymbol{P}(\boldsymbol{0})$ respectively to accomplish circular trace in task space. The simple transformation is given in equation 4.

2.3 Load sharing while tracing

Gripping the load securely while moving is one of the common applications in the hot cell. In robot cooperation scenario, this is visualized as the end-effectors sharing a common point in the task space. The figure 4 shows the task of gripping and moving the load. The simple way to achieve this is by mirroring $\mathbf{O_2}$ about axis $\mathbf{X_M}$ (so as to make $\mathbf{Z_1}$ and $\mathbf{Z_2}$ collinear) and translate, $\mathbf{O_2}$ such that the robot 2 is offset along $\mathbf{Z_1}$ and mounted parallel but offset to robot 1. The end effector reference points are not coinciding but have an offset along $\mathbf{Z_1}$ and that is acceptable for many load sharing applications. The above method is simple as the same motion file works for both the robots and provides easier insight into how the robots cooperate, as both the task space and joint space motion are identical. However, this involves disturbing the setup and is not admissible. The other option requires no disturbance to the setup but requires certain transformations to be performed on the input data file generated for robot 1. Care should be taken that the physical understanding or insight into the nature of cooperation is not lost.

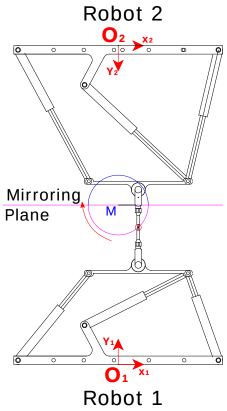

Figure 4: Two robots sharing the load while in traverse

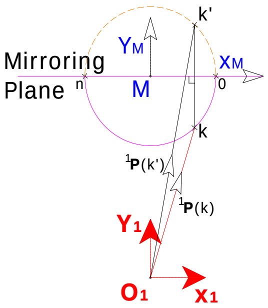

Figure 5: Task point and its mirror point description for Robot 1

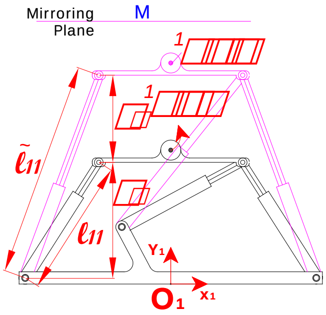

Figure 6: Configuration Transformation from robot 1 to 2

The circular task space trajectory is divided into two semi-circular trajectories about the mirroring plane as in case 1. The parameter description is given in figure 5. For every point of $^{1}\boldsymbol{P}(\boldsymbol{k})$ on the semicircle and its mirror point, $^{\sim}\boldsymbol{P}(\boldsymbol{k}),\ \boldsymbol{k}\ \boldsymbol{\epsilon}\ [\boldsymbol{0},\boldsymbol{n}]$ an IKP solution is obtained. The IKP solution for $^{\sim}\boldsymbol{P}(\boldsymbol{k}), \boldsymbol{k}\ \boldsymbol{\epsilon}\ [\boldsymbol{0},\boldsymbol{n}]$ is obtained as a function of IKP solution of $^{1}\boldsymbol{P}(\boldsymbol{k}), \boldsymbol{k}\ \boldsymbol{\epsilon}\ [\boldsymbol{0},\boldsymbol{n}]$ and is given below

$$^{m}\boldsymbol{P}(\boldsymbol{k}) = {}^{m}\boldsymbol{P}(\boldsymbol{k})\hat{\boldsymbol{\imath}} + {}^{m}\boldsymbol{P}(\boldsymbol{k})\hat{\boldsymbol{\jmath}} \quad ; \qquad {}^{m\sim}\boldsymbol{P}(\boldsymbol{k}) = {}^{m}\boldsymbol{P}(\boldsymbol{k})\hat{\boldsymbol{\imath}} - {}^{m}\boldsymbol{P}(\boldsymbol{k})\hat{\boldsymbol{\jmath}}$$

$$^{1}\boldsymbol{P}(\boldsymbol{k}) = \boldsymbol{O_1M} + {}^{m}_{1}\boldsymbol{R}\,{}^{m}\boldsymbol{P}(\boldsymbol{k})\,\hat{\boldsymbol{\jmath}} \quad ; \qquad {}^{1}\boldsymbol{P}(\boldsymbol{k}') = \boldsymbol{O_1M} - {}^{m}_{1}\boldsymbol{R}\,{}^{m}\boldsymbol{P}(\boldsymbol{k})\,\hat{\boldsymbol{\jmath}}$$

The IKP solution for $^{1}\boldsymbol{P}(\boldsymbol{k})$ can be determined and are given as l_{11}, l_{21} and l_{31}. The ikp solution for $^{\sim}\boldsymbol{P}(\boldsymbol{k})$ can be obtained in terms of l_{11}, l_{21} and l_{31} as follows (see figure 6)

$$l_{\tilde{1}1} = \sqrt{l_{11}^2 + s_m^2 + 2s_m s_1} \quad ; \qquad l_{\tilde{2}1} = \sqrt{l_{21}^2 + s_m^2 + 2s_m s_2} \quad ;$$
$$l_{\tilde{3}1} = \sqrt{l_{31}^2 + s_m^2 + 2s_m s_3}$$

$l_{12} = l_{\tilde{1}1}$; $l_{22} = l_{\tilde{2}1}$ $l_{32} = l_{\tilde{3}1}$

The solution set for the point, ${}^{\mathbf{1}}\boldsymbol{P}(\boldsymbol{k})$ serve as a input data file for robot 1 and the second solution set for mirror point, ${}^{\sim}\boldsymbol{P}(\boldsymbol{k})$ serve as a input data file for robot 2 as described in figure 2.

3 Special Application through Mirror Motion with Two Robots in Cooperation

A single robot to realize an ultra slow uniform motion at the end-effector is not feasible because the motors exhibit jerky motions at very slow speeds. Two cooperative robots are set to move near around most proficient speeds, yet the relative motion between their end-effectors can be programmed to have ultra slow velocity. Certain applications in the hot cell require a smooth relative motion between two objects. A common application in the hot cell is laser cutting. Often, ultra slow speed is required between the laser beam and the job being cut. The typical relative velocity between the objects varies depending upon the intensity of the laser and the material composition and thickness. In cooperative mode, the laser beam is manipulated by robot 1 and the job which has to be cut is manipulated by robot 2. The arrangement of co-operative robots is as discussed in section 2. To realize ultra slow uniform velocity at the interface of the beam and the job, both beam and the job are set to motion along the same direction. It is like a laser beam following the job along the trajectory. Case 3 discussed in subsection 2.3 can be utilized to obtain such a motion between the cooperating robots.

Let the task space velocity of robot 1 and 2 be T_1 and T_2 respectively. If the task space ultra slow velocity between the interface of the beam tip and the job is ΔT. The CPRI code should achieve $(T_1 - T_2) = \Delta T$;

The task space velocities of robot 1 and 2 in terms of the components are written as
$\{T_1\} = [v_{x1} \quad v_{y1} \quad \omega_1]^T$; $\{T_2\} = [v_{x2} \quad v_{y2} \quad \omega_2]^T$

The Jacobian, relating the task space velocity to the leg velocity for robot 1 and 2 are given as

$$\{V_1\} = [\,\mathrm{J}_1\,]^T\{T_1\}\,; \qquad \{V_2\} = [\,\mathrm{J}_2\,]^T\{T_2\}$$

where, $\{V_i\} = [v_{1i} \quad v_{2i} \quad v_{3i}]^T$ is the vector containing leg velocities of robot i [i=1,2]. $[\,\mathrm{J}_1\,]$ and $[\,\mathrm{J}_2\,]$ are the 3×3 Jacobian matrix of robot 1 and 2 respectively The relationship between $\{V_1\}$ and $\{V_2\}$ can be obtained as discussed in section 2.

4 Experimental Results

Two prototype planar parallel robots are setup in mirror configuration as described in section 2. Figure 7 gives the experimental result of the repeatability test and the repeatability values typically represent the manipulator precision in trajectory following. Many experiments have been conducted to see two robots participating to complete the given task in a cooperative manner. The purpose of the experiments is to understand the simple ways to bring two robots to collaborate to conduct a task in a hot cell. Many experiments discussed in section 2 have been conducted. In all the experiments, the robots manipulate the pen and the writing pad. Figure 8 shows the application analogy of the cases discussed in section 2.

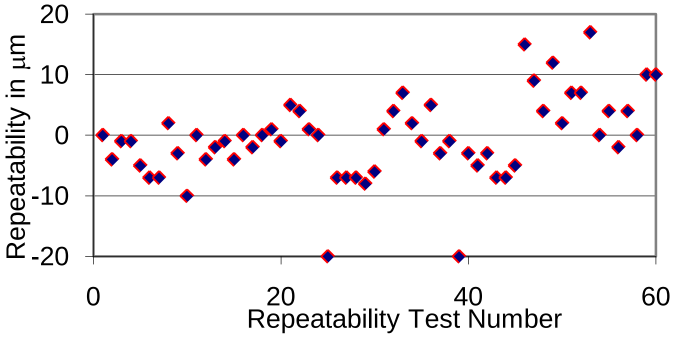

Figure 7: Precision in following a trajectory

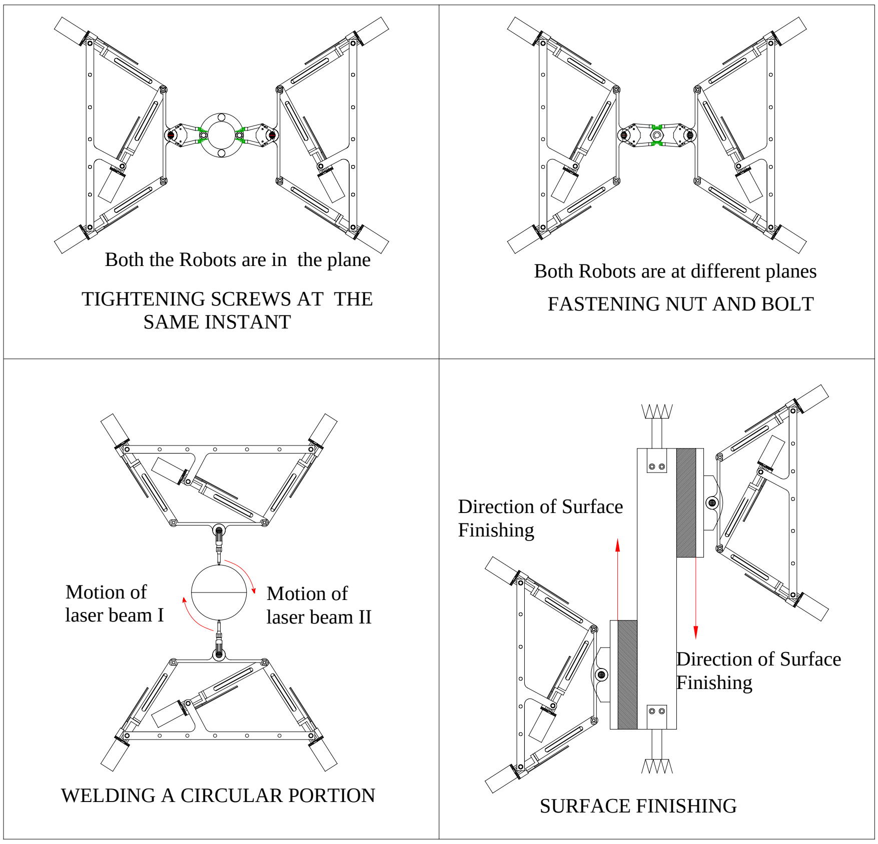

Figure 8: Applications using 2 Parallel Planar Robots in Mirror Mode

Figures 9 and 10 show the snap shot of two of the many experiments conducted in cooperative mode.

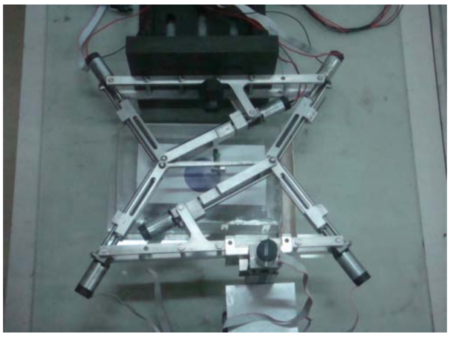

Figure 9: An Arrangement of Planar Parallel Robots in a cooperation mode

Figure 10: Planar Parallel Robots performing a Common Task in cooperation

5 Conclusion

The context of the development is to automate several operations that take place inside the nuclear hot cell. The paper explained the concept of mirroring manipulation considering two cooperating planar parallel robots. The type of cooperation is categorized and analyzed to show simple ways to achieve the collaboration. Several applications that can be programmed under this framework are discussed. The CRPI code can be implemented in all the three types of control. The experiments have revealed that the mirror manipulation scheme and CPRI code simplify the planning and give a better insight into the role of the participating robots.

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