An Optimization Approach for Path Synthesis of Four-bar Grashof Mechanisms

A.S.M.Alhajj, J.Srinivas

Abstract

This paper presents an optimization scheme based on the principle of harmony-search for path synthesis of Grashof four-bar mechanisms. The objective in this work is to minimize position error defined by the coordinates of coupler point subjected to satisfaction of constraints such as Grashof criterion and sequence on input link angles in addition to geometrical constraints on the design variables. A generalized approach is formulated such that the minimization of objective is carried-out only after a feasible solution has been obtained. Two benchmark examples for path synthesis with and without prescribed timings (input link angles for each precision point) are considered to illustrate the effectiveness of the method.

Keywords: Path synthesis, Position error, Crank-rocker mechanism, Constrained-Optimization, Harmony search.

1 Introduction

Linkages having rigid members are exclusively used in the area of mechanical engineering for motion and energy transmission from one or more input members to output members. Four-bar linkages are a class of simple but practically important mechanisms. Their utilization ranges from simple devices, such as windshield-wiping mechanisms and door-closing mechanisms to complicated rock crushers, sewing machines, round balers and suspension systems of automobiles. Two basic concepts involved in the design of linkages are: analysis and synthesis. The term synthesis refers to the process of obtaining linkage parameters to obtain a required task. In dimensional synthesis of linkages, three different problems are commonly seen. These include: motion-generation, function-generation and path generation.

Design of a linkage for generation of a particular path is relatively a difficult task. In fact, the problem of path synthesis of a four-bar linkage is to generate a mechanism whose coupler point can trace the desired trajectory or target points. The path synthesis of a four-bar linkage has been actively studied during the past 50 years. There has been a large number of studies on this topic using a variety of methods. Analytical solution to the general problem of four-bar linkage synthesis with more than five precision points is a quite difficult task. For such situations, a variety of numerical methods can be employed. One such approach is optimization, in which a defined objective function in terms of linkage variables is minimized under certain constraints. The most common objective function is position error,

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defined as the sum of the squares of the Euclidean distance between the target and generated coupler points.

Several authors described the use of unconventional optimization schemes for solving path synthesis problem of four-bar mechanisms. Cabrera *et al*. [1] initiated the application of genetic algorithms (GA) for optimal synthesis of four-bar linkages. Later-on, evolutionary algorithms [2], Genetic-fuzzy scheme [3], geometric constrained programming approach [4], ant-gradient algorithm [5], particle swarm optimization (PSO) and differential evolution [6], real-coded evolutionary algorithms [7] have been adopted for path synthesis problem. Some works [8-9] highlighted the use of meta-heuristic methods for multi-objective Pareto optimum synthesis of four-bar linkages. Recently, there is a gaining interest towards development of new optimization heuristics that generate the outputs relatively faster and accurate with less number of input parameters. Harmony search optimization is one such approach finding its applications in several engineering problems. Implementation of harmony search optimization (HSO) method in mechanism synthesis has relative merits over several existing optimization schemes.

2 Basic Synthesis Procedure

In path generation problem of four-bar mechanism, synthesis procedure is accomplished by some precision points to be traced by the coupler point P of the mechanism as shown in Figure 1.

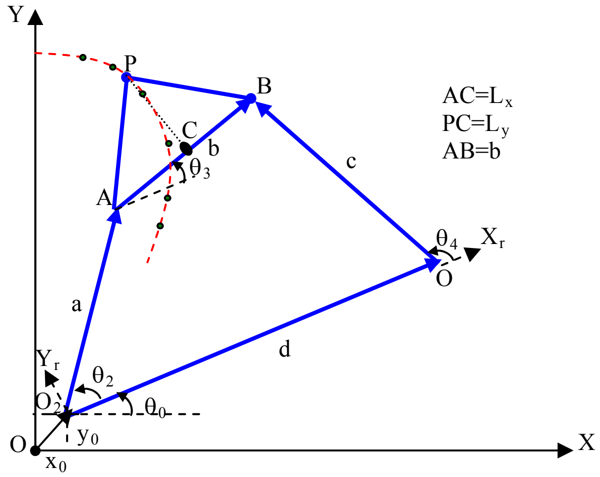

Figure 1: Four-bar mechanism in global coordinate frame

An optimization scheme may be adopted to find the set of dimensional parameters of the mechanism, namely link lengths (a, b, c, d, L_x, L_y etc) and input link angles (θ_2), so that the error between the precision points (representing the desired trajectory) and the actually traced points by the coupler is minimized. While minimizing the error function, there are number of constraints such as the Grashof condition, decreasing or increasing sequence of input link angles and the range of design variables have to be satisfied. This forms a multivariable, constrained-nonlinear optimization problem. Here, the objective function called tracking error (TE) is evaluated from the traced points (P_x^i, P_y^i), (where i=1,2…,N) defined with respect to global coordinate frame XOY. From Fig.1, the position vector of the coupler point P in reference frame $X_rO_2Y_r$ can be expressed as a vector equation:

$$\vec{r}^{\,P} = \vec{a} + \vec{L}_x + \vec{L}_y \tag{1}$$

which can be expanded as:

$$\begin{Bmatrix} P_{xr} \\ P_{yr} \end{Bmatrix} = a\begin{Bmatrix} \cos\theta_2 \\ \sin\theta_2 \end{Bmatrix} + L_x\begin{Bmatrix} \cos\theta_3 \\ \sin\theta_3 \end{Bmatrix} + L_y\begin{Bmatrix} -\sin\theta_3 \\ \cos\theta_3 \end{Bmatrix} \tag{2}$$

Here, the coupler link angle θ_3 is computed using the following vector loop equation of the four-bar mechanism:

$$\vec{a} + \vec{b} - \vec{c} - \vec{d} = 0 \tag{3}$$

This can be expressed in its components with respect to relative coordinate frame $X_rO_2Y_r$ as:

$$a\cos\theta_2 + b\cos\theta_3 - c\cos\theta_4 - d = 0 \tag{4}$$

$$a\sin\theta_2 + b\sin\theta_3 - c\sin\theta_4 = 0 \tag{5}$$

After elimination of θ_4, the unknown angle of θ_3 is computed for known values of input link angle θ_2. This takes the following form known as Freudenstein's equation:

$$K_1\cos\theta_3 + K_2\cos\theta_2 + K_3 = \cos(\theta_2-\theta_3) \tag{6}$$

where K_1=d/a, K_2=d/b and $K_3=\dfrac{c^2-d^2-a^2-b^2}{2ab}$ (7)

From this equation, the two solutions (open and crossed) can be written as:

$$\theta_3^{1,2} = 2\tan^{-1}\left(\frac{-E \pm \sqrt{E^2-4DF}}{2D}\right) \tag{8}$$

where

$D=\cos\theta_2 - K_1 + K_2\cos\theta_2 + K_3$, $E=-2\sin\theta_2$ and $F=K_1+(K_2-1)\cos\theta_2 + K_3$ (9)

Finally, the position of coupler P, with respect to global or world coordinate system XOY is defined by:

$$\begin{Bmatrix} P_x \\ P_y \end{Bmatrix} = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix}\begin{Bmatrix} P_{xr} \\ P_{yr} \end{Bmatrix} + \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} \tag{10}$$

The coordinates are used to define the following objective function:

$$\text{Minimize TE} = \sum_{i=1}^{N}[(P_{xd}^i - P_x^i)^2 + (P_{yd}^i - P_y^i)^2] \tag{11}$$

where N is number of precision points specified on the desired path and (P_{xd}^i, P_{yd}^i) are given set of desired precision point coordinates. For effective functioning of linkage, one or several constraints on the dimensions are often posed. In this work, the following constraints are considered:

(1) Range for design variables: The magnitudes of link lengths and coupler point coordinates as well as joint angle ranges are restricted between a low and high value. These are called side or geometric constraints.

(2) Grashof criterion: This is the requirement that the input link of linkage as a crank and the mechanism is either crank-rocker or drag-link mechanism. This condition is written as:

$$\frac{2(x_{min} + x_{max})}{(a + b + c + d)} - 1 \leq 0 \tag{12}$$

where x_{min} and x_{max} are respectively the minimum and maximum values of link lengths: a,b,c and d. If this condition is not satisfied, a new design vector is selected.
(3) Order of crank angles: As there is a possibility of a large combination of mechanisms that would generate same coupler curve, we need to pose either clockwise or anticlockwise rotation constraint on crank angle. This is especially important while generating paths without prescribed timings, where the angle of crank for each coupler point is also of concern. This increases the size of design vector by including as many crank angles as the number of given precision points. If $\theta_2^1, \theta_2^2, \ldots.. \theta_2^N$ are the required crank angles, the order constraint is written as:

$$\text{sign}(\theta_2^{i+1}-\theta_2^{i}) = \text{sign}(\theta_2^{2}-\theta_2^{1}) \text{ for all } i=2,3,\ldots,(N-1) \tag{13}$$

Here sign is sign function defined as sign(θ)=+1 if θ≥0, otherwise sign(θ)=-1.If this condition is not meeting, the solution is rejected and new set of random variables are selected satisfying the above two conditions.

For handling the objective function with all these constraints, a dynamic objective function approach is adopted in this paper. The original problem is converted into following unconstrained bi-objective optimization problem:

$$\text{Min (CV, TE)}, \; X=\{x_1,x_2,\ldots x_n\} \subset S \in R^n \tag{14}$$

where CV is constraint violation if any and $S \in R^n$ is the search space defined by parameter bounds. Hence, we could minimize TE only when CV becomes zero. When the set of variables lies outside the feasible region (CV>0), it is not necessary to calculate objective function TE, resulting in reduction of computation cost.

3 Proposed Optimization Scheme

The Harmony Search (HS) algorithm, compared to other optimizing methods like genetic algorithms, is very simple in idea and involves very few setting parameters as well as easy to execute. HS method was developed originally by Geem *et al.* [10] and later on several modifications have been suggested to improve its performance. The basic HS method applies the musical procedure of seeking for the best state of harmony. The harmony in music is similar to solution vector in the optimization problem and musician's seeking for best harmony is comparable to global search system in this optimization method. The basic steps of the approach are as follows:
Step 1: Initializing the problem and algorithm parameters: In this step, the optimization problem is defined in terms of decision variables X and effective objective function f(X). The parameters of algorithm are also designated in this step. These are the harmony memory size (HMS) or the number of solution vectors in the harmony memory; harmony memory considering rate (HMCR); pitch adjustment rate (PAR); number of decision variables (N) and the number of improvisations (NI) or stopping base. The harmony memory (HM) is a memory location where all the solution vectors are stored. This HM is similar to the genetic pool in the genetic algorithm. The HMCR and PAR are parameters used to enhance the solution vector and are defined later in Step 3.

Step 2: Initializing the harmony memory: The HM matrix is initially filled with as many randomly generated solution vectors as the HMS, as well as with the corresponding function values of each random vector, f(X).

$$\mathrm{HM}=\begin{bmatrix} x_1^1 & x_2^1 & . & x_N^1 & | & f(X^1) \\ x_1^2 & x_2^2 & . & x_N^2 & | & f(X^2) \\ . & . & . & . & | & . \\ . & . & . & . & | & . \\ x_1^{HMS} & x_2^{HMS} & . & x_N^{HMS} & | & f(X^{HMS}) \end{bmatrix} \tag{15}$$

Step 3: Improvising a new harmony: New Harmony vector, $X'=\{x_1', x_2', \ldots, x_N'\}$ is created from the HM based on the memory considerations, pitch adjustments and randomization. For example, the value of first variable x_1' in new harmony-vector is created from any value in specified HM range. The values of other variables are also selected likewise. The HMCR, which varies between 0 and 1, is the rate of choosing one value from the historical values stored in HM, while (1-HMCR) is rate of randomly selecting one value from the possible range of values. Every component obtained by memory consideration is examined for condition of pitch-adjustment. This operation uses the PAR parameter, in which the pitch is adjusted as:

Pitch adjusting decision for x_i'= yes, with probability PAR

=no, with probability 1-PAR (16)

When the pitch adjustment decision for x_i' is yes, x_i' should be updated as: $x_i'\pm bw\times rand(\ )$, where $bw=bw_{max}\ \exp(c\times gn)$ with $c=\log_e(bw_{min}/bw_{max})/NI$, is an arbitrary distance bandwidth in each generation *gn*, while rand() is a random number between 0 and 1. The constants bw_{min} and bw_{max} are minimum and maximum values of bandwidth respectively. The HMCR and PAR parameters help the algorithm to find globally and locally improved solutions, respectively. PAR and bw in the HS algorithm are found to be fine-tuning parameters of optimized solution vectors. In present task, just like bandwidth, adaptive PAR values for each generation are employed. That is, PAR for generation gn in terms of minimum and maximum pitch adjustment rates is:

$$PAR=PAR_{min} + (PAR_{max}-PAR_{min})\times gn/NI \tag{17}$$

Step 4: Updating the harmony memory: In this step, if the new Harmony vector X′ is better than the worst harmony in the HM in terms of the objective function value, the new harmony is included in the HM and the worst existing harmony is excluded from the HM. The HM is then sorted by the value of objective function.

Step 5: Stopping criterion: When the maximum number of improvisations (NI) has reached, the computation is terminated. Otherwise, steps 3 and 4 are repeated.

4 Numerical Simulations

The HS optimization scheme is implemented in MATLAB environment. The program employs two sub-functions for handling constraints and objective. Two cases are described here for path synthesis problem, one without and other with prescribed timings.

Case 1: Path generation without prescribed timing, 10 target points: The problem was proposed by Acharya and Mandal [6] and is defined as:

–Design variables (19 variables):

$$X = [a, b, c, d, L_x, L_y, \theta_0, x_0, y_0, \theta_2^1, \theta_2^2, \ldots, \theta_2^9, \theta_2^{10}]$$

–Target points (Ten points):

$\{P_d^i\}$={(20,10),(17.66,15.142),(11.736,17.878),(5,16.928), (0.60307,12.736), (0.60307,7.2638), (5, 3.0718),(11.736,2.1215),(17.66,4.8577),(20,10)}

- Limits of the variables:

$a, b, c, d \in [5,80]$; $L_x, L_y, x_0, y_0 \in [-80, 80]$; $\theta_0, \theta_2^1,\ldots, \theta_2^{10} \in [0, 2\pi]$

Figure 2 shows the ten target points and the coupler curve obtained using the harmony search method for this case. The time taken to run the program is just 10.09 seconds for 15,000 cycles. The convergence plot for this case is illustrated in Figure 3.

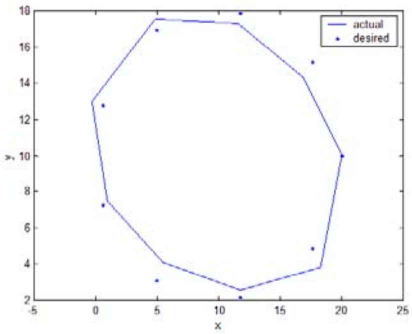

Figure 2: Coupler curve for case-1

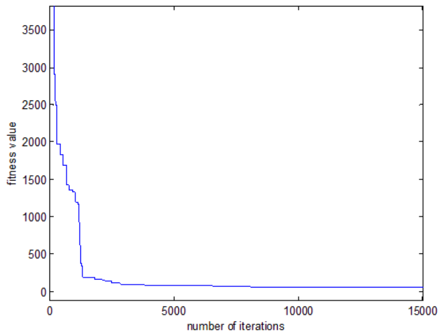

Figure 3: Fitness-convergence plot

The synthesized geometric parameters and the corresponding values of the mean error are shown in Table 1, together with the available results.

Table 1 Design variables obtained and comparison with available data

Variable	GA [6]	PSO[6]	GA-DE[7]	HSO
a	9.109993	8.687	8.24689	8.3195
b	72.93651	36.155	45.8968	75.9046
c	80	80	58.5404	64.5646
d	79.98151	52.535	80	59.8899
L_x	0	0	-6.40389	-21.3858
L_y	0	1.481	-9.12264	19.0138
x_0	10.15597	11.0021	6.52409	-6.9700
y_0	10	11.0955	20.522	-11.4564
θ_0	0.026149	1.4035	0.136532	5.5185
θ_2^1	6.283185	6.2826	6.05991	1.0676
θ_2^2	0.600745	0.6153	0.488453	1.7156
θ_2^3	1.372812	1.3054	1.17805	2.3344
θ_2^4	2.210575	2.188	1.88339	3.0524
θ_2^5	2.862639	2.913	2.59806	4.0346
θ_2^6	3.420547	3.4993	3.28585	4.6542
θ_2^7	4.072611	4.1255	3.96674	5.1860

θ_2^8	4.910373	4.9199	4.65966	5.7453
θ_2^9	5.68244	5.685	5.35231	6.2830
θ_2^{10}	6.283185	6.2833	6.06263	6.2832

The tracking error is found to be 2.152 as against 2.281 in GA [6] for this case.

Case2: Path generation with prescribed timing, 18 target points:

This problem requires generating a path along the 18 points and is proposed by Kunjur and Krishnamurthy (KK) [11]. It is defined as follows:

Design variables (10 variables): $X=[a, b, c, d, L_x, L_y, \theta_2^1, X_0, Y_0, \theta_0]$

-Target points: $\{P_d^i\}=\{(0.5,1.1),(0.4,1.1),(0.3,1.1),(0.2,1.0),(0.1,0.9),(0.005,0.75),(0.02,0.6),(0,0.5),(0,0.4),(0.03,0.3),(0.1,0.25),(0.15,0.2),(0.2,0.3),(0.3,0.4),(0.4,0.5),(0.5,0.7),(0.6,0.9),(0.6,1)\}$

$\{\theta_2^i\}=\{\theta_2^1, \theta_2^1+20^o * i\}$, (where i=1,2,…17)

-Limits of the variables:

$a, b, c, d \in [5,50]$; $L_x, L_y, x_0, y_0 \in[-50, 50]$; $\theta_0, \theta_2^1 \in[0, 2\pi]$

Table 2 gives the parameters of the optimal mechanism generated by HSO algorithm and compares them with results reported in literature. The results show that the HSO solution gives the smallest average error.

Table 2 Comparative results for case 2

Variable	GA-DE[7]	Cabrera[1]	KK[18]	HSO
a	0.218612	0.237803	0.36355	0.2547
b	42.4842	4.828954	2.91374	28.5878
c	32.747	2.056456	0.49374	20.3845
d	49.9592	3.057878	2.85452	40.3277
L_x	-47.966	0.767038	1.031223	-17.9087
L_y	15.3586	1.850828	1.717471	-10.5145
x_0	44.1758	1.776808	0.95928	17.1878
y_0	-23.9643	-0.64199	-1.19645	-11.3451
θ_0	5.37543	1.002168	0.76398	4.6397
θ_2^1	1.88551	0.226186	1.2756	2.4627
Error	0.04784	0.0421	0.0226	0.01836

Fig.4 shows the coupler curve generated by the optimal mechanism using proposed HSO method along with that of GA-DE approach [7].

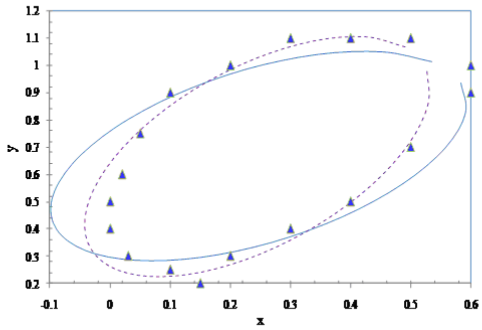

Figure 4: Coupler curve for case 2 (solid: HSO, dotted: GA-DE [7], Δ: Desired)

In both the cases, the algorithm parameters adopted are: NI=15000, HMS=30, HMCR=0.95, PAR_{max}=0.9, PAR_{min}=0.4, bw_{min}=0.0001, bw_{max}=1. The simulations are conducted using X-86 based PC with Intel core-2 Duo, 3.0 GHz processor.

5 Conclusions

This paper presented the results of path synthesis of four-bar mechanism using harmony search optimization method. Path tracking error was defined as objective function and constraints like variable bounds, Grashof criterion and angle sequence at input link were imposed. The algorithm resulted in near optimal solutions faster with acceptable accuracy. The methodology may be extended for path synthesis of higher order linkages (6-bar planar parallel mechanisms) as well as to obtain dimensions of hybrid (to achieve both the desired path and motion) mechanisms.

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