Link Geometry Synthesis for Prescribed Inertia

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Abstract

In this paper, a methodology is presented for synthesis of planar link's geometry for prescribed inertia parameter values subject to given geometric constraints and having the flexibility of varying topology if desired. Synthesis starts with an initial geometry that satisfies the geometric constraints but not the inertial requirements. As the geometry evolves, topology has to change according to the geometric constraints. The challenge here is to have a controlled change of topology to avoid splitting of domain into multiple components. A *novel method of achieving control over the topological evolution* of the domain is identified in this work using two types of transformations namely domain preserving transformation and domain splitting transformation. Link geometry is represented as a set of non-intersecting closed polygons consisting of a set of vertices and directed edges (loops), one of which is the outer boundary and the others are holes. The desired inertial properties of the domain are achieved through iterative gradient based optimization.Results obtained from kinematic synthesis provide joint locations, outcome of link geometry synthesis for interference free motion provides the allowable domain; these together form the set of geometric constraints. The results of dynamic synthesis provide the optimal (target) values of inertial parameters for the present geometry synthesis procedure. The methodology presented in this work enables exploration of multiplicity of solutions.

Keywords: Geometry Synthesis, Dynamic Balancing, Shape Optimization

1 Introduction

Engineering design of a mechanism is incomplete without the detailed geometry of its links. During kinematic analysis and synthesis, a mechanism is usually represented by a simplified schematic representation and the true shape of the links is not of concern. On the other hand, the inertial parameters of the links, viz., mass, centre of mass and moment of inertia essentially depend on the geometric shape of the links and dictate the dynamic characteristics, viz. shaking force, shaking moment, driving torque, time of travel, etc. of the mechanism. As reported by [5], mechanism balancing problem is old and well defined, one that aims at minimizing the effects of shaking force and shaking moments. Most of the techniques found in literatureare based on parametric mass distribution [1, 9], counter-weight addition [6] or use of additional links [10]. *The dynamic synthesis procedures typically determine location of the point masses; the inverse problem of equivalent point-mass to link shape is unaddressed in literature.* The subject of shape and topology synthesis aims to determine the explicit shapes using finite elements methods [8]. Expansion of its scope to satisfaction of instantaneous motion characteristics has led to the concept of compliant mechanisms. The techniques are not suitable for synthesis for inertial performance due to the local nature of the structural responses and the global

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nature of the inertial characteristics. Both grid and front propagation (level set) based techniques [7] leads to uncontrolled change in topology of the domain which is unacceptable for the present problem *because a set of unconnected domains which satisfy inertial requirements are practically meaningless*. Procedures exist for efficient determination of simple geometric shapes for the links of a mechanism, primarily aiming towards interference free motion, which results in identifying a feasible material domain for each link's geometry [4]. However, the effect of the result geometry on the dynamics of the mechanism is not considered. Thus it is necessary to explore link shapes that have the necessary inertial properties and still are non-interfering.

The work presented proposes an optimization based methodology for the synthesis of link's geometry where a given initial shape of a link is systematically modified such that its inertial properties match the desired values without losing its geometric connectedness. Kinematic dimensions and link-interference characteristics are preserved through suitable constraint modelling. This is achieved through the development of novel topology control transformations.

2 Link's Geometry Representation

Link's geometry is represented as a set of non-intersecting closed polygons (loops), one of which is the outer boundary and the others are holes in the component. Each polygon is a set of directed edges connecting the vertices (points on the boundary contours) in such a manner that material is on the left when travelled along the edge in the established direction. Fig. 1 shows an illustration of this representation. Each vertex $\boldsymbol{V_i}$ is given an index and is located by two coordinates $(\boldsymbol{x_i}, \boldsymbol{y_i})$ with i being the index value. Each edge $\boldsymbol{E_i}$ connects two of the vertices $\boldsymbol{V}_\mathrm{i}$ and $\boldsymbol{V_{i+1}}$.

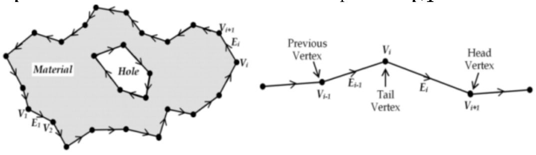

Figure 1: Vertex-edge representation of link's boundary contours

3 Topology Evolution

Geometry synthesis starts with an initial geometry that *satisfies the kinematic and geometric constraints but not the inertial requirements*. The geometry of a link can have arbitrary topology but has to be a single, potentially multiply connected, domain. As the geometry evolves during synthesis, topology could change within the limits of geometric constraints. In structural topology optimization using level set methods, it is observed that the single component connectedness of the material domain is ensured by the underlying field variables which influence the objective function during optimization. There is no geometry dependent physical variable which affects the link's inertia parameters. Hence the procedures developed for structural optimality does not suit the requirements of link's geometry synthesis for inertia requirements. The primary challenge here is to have a controlled change of topology to avoid splitting of domain into multiple components.

Computation of inertia parameters requires that the boundary contours do not intersect. During geometric evolution, boundary contours come close to each other and tend to intersect each other resulting in either merging or splitting of domains. The following paragraphs present two *geometric transformations, hitherto unavailable in literature, that enable local geometric test for global topological inference*.

3.1 Domain preserving transformation (DPT)

Fig.2 (a) shows a material domain bounded by a single directed contour $\boldsymbol{C_1}$, such that material is onto the left when traversed along this curve. Region $\boldsymbol{M}$ indicates that boundary contour is too close to itself at a certain step of geometry modification. Under such situation, the boundary is modified locally as shown in the Fig. 2(a) which results in the creation of a hole. This transformation is termed as *domain preserving transformation* (DPT), because the material domain still remains as a single component though it gets multiply connected. It can be observed that the edge-swapping process in effect adds a small amount of material to the body. For the polygonal domains the transformation is done as follows. Fig. 2(b) shows a coarse vertex-edge representation using 21 edges (or vertices), where vertices $\boldsymbol{V_5}$ and $\boldsymbol{V_{13}}$ are close to each other. This indicates that a rearrangement of the edges attached to these vertices will change the topology. There are two sets of neighbouring vertices to $\boldsymbol{V_5}$ and $\boldsymbol{V_{13}}$. If vertices $\boldsymbol{V_6}$ and $\boldsymbol{V_{12}}$ are closer than $\boldsymbol{V_4}$ and $\boldsymbol{V_{14}}$, the altered vertex-edge connectivity would be as in Fig. 2(c); otherwise, it would be as in Fig. 2(d).

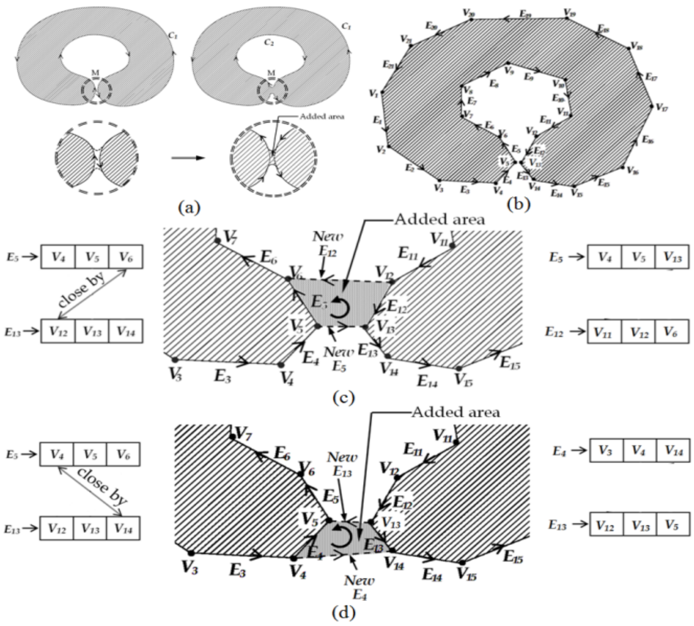

Figure 2: (a) Domain preserving transformation, (b) Vertex-edge representation of boundary curves, (c) (d) two possible states after transformation

3.2 Domain splitting transformation (DST)

Fig.3 (a) shows a material domain bounded by a single directed contour $\boldsymbol{C_1}$, such that material is onto the left when traversed along this curve. Region denoted by $\boldsymbol{N}$ indicates that boundary contour is on the verge of self-intersection. The edge swapping operation, as mentioned above, splits the domain into multiple components, if the boundary is a single contour. Hence this transformation is termed as *domain splitting transformation* (DST).Contrary to DPT, a small patch of area is removed from the original domain during DST. Fig. 3(c) and 3(d) show the two alternate changes to a polygonal B-rep of the link geometry.

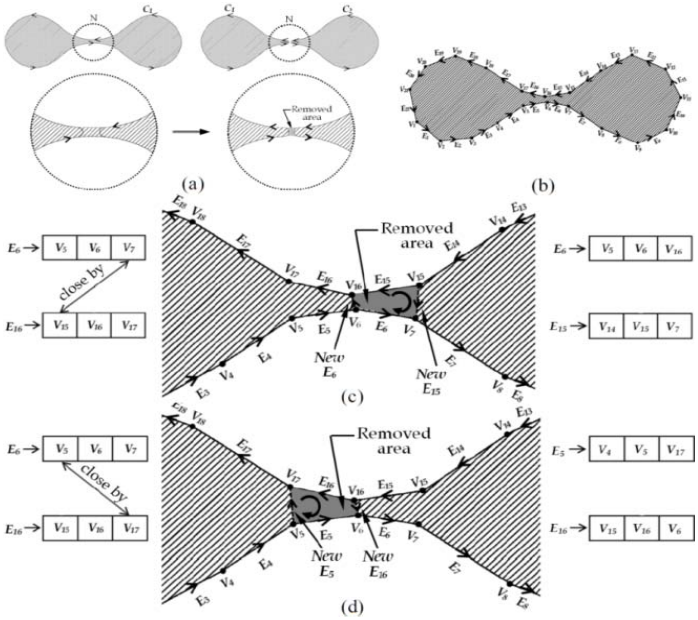

Figure 3: (a) Domain splitting transformation, (b) Vertex-edge representation of boundary curves, (c) (d) two possible states after transformation

3.3 Transformations of multiply connected domains

When a domain is multiply connected, it would have multiple boundary curves (loops). In that case, if DST is applied to segments from different loops, it merges the two loops into one; it does not split the domain and the number of holes reduces by one. This is an acceptable change. But DST applied on segments of the same boundary curve, the domain splits. Because of the edge based data structure, loop is implicit in the representation; for the sake of efficiency, the loop-index for every edge is maintained in the data structure and appropriately altered when such a case for DST arises.

4 Geometry Design Procedure

The process of geometry design starts with an initial geometry which satisfies the constraints but does not have desired inertial characteristics. The boundary contours of this geometry shall evolve through an optimization procedure that minimizes the difference between the link's inertia parameters with that of the desired ones. Iterative gradient based optimization is used in the present work. Alternate methods can also be used.

4.1 Design variables

Consider a closed contour $\boldsymbol{C} \in \mathrm{R}^2$ evolving in space. The evolution of $\boldsymbol{C}$ with respect to time can then be described by the following equation.

$$\frac{\partial \boldsymbol{C}}{\partial t} = \alpha \vec{\boldsymbol{t}} + \beta \vec{\boldsymbol{n}} \quad (1) \qquad \frac{\partial \boldsymbol{C}}{\partial t} = \bar{\beta} \vec{\boldsymbol{n}} \quad (2)$$

where$\vec{\boldsymbol{t}}$ is the tangent, $\vec{\boldsymbol{n}}$ is the outward normal, α and β are arbitrary functions describing the tangential and the normal speeds of the contour. It is known from [3] that for each choice of speed functions (α, β) there exist other speed functions $(0,\bar{\beta})$ such that the resulting contour shapes are equivalent. The tangential component, α, affects only the parameterization of the contour while β changes the contour's shape and Eq. (1) can be simplified to Eq. (2).This means contour evolves only in normal direction. In vertex-edge representation of the contour, each vertex of the contour is allowed to move in the normal direction which is computed from the two neighbouring vertices, thus resulting in change of shape and hence a change in the inertial parameters. The amount of movement for each vertex in their corresponding normal direction is governed by $\bar{\beta}$and hence it gives the set of design variables. For the polygonal contours, the line passing through the current vertex and perpendicularto theline joining its adjacent vertices has been taken here as the estimated normal direction. An initially smooth contour may become rough. Over a number of iterations this effect often gets accentuated leading to theoretically valid but practically unusable shapes. To overcome this problem a *moving average filter* is applied after every fixed number of iterations for reducing irregularities in the contour. This changes the inertial properties, but it gets compensated with the progress of optimization.

4.2 Objective function and constraints

The objective of the present endeavour is to minimize the difference between the inertial parameter values of the evolving contour with respect to the desired ones. The parameters of relevance here are *mass* (or area, A), coordinates of *centre of mass* (CG) and polar moment of inertia (I) about CG. Each link is substituted with dynamically equivalent two-point mass model whose parameters are equivalent area (assuming constant thickness of each link) and coordinates of the points i.e. (A_i, x_i, y_i) with *i=1, 2*. Three (A_1, A_2, x_1) of the six parameters are assigned as in Eq. (3-4); the other three (y_1, x_2, y_2) are determined from the equivalence relations given in Eq. (5-7), where $k_G = \sqrt{\frac{I}{A}}$ is the radius of gyration.

$$A_1 = A_2 = \frac{A}{2} \quad (3) \qquad x_1 = \frac{CG_x}{2} \quad (4) \qquad y_1 = CG_y + \sqrt{(k_G^2 - (\frac{CG_x}{2})^2)} \quad (5)$$

$$x_2 = \frac{3CG_x}{2} \quad (6) \qquad y_2 = 2CG_y - y_1 \quad (7)$$

These values of the two-point mass parameters are used in objective function evaluation as shown in Eq. (8) which is the sum of square of the percentage of error in each individual parameter value, where (y_1^*, x_2^*, y_2^*) are the point mass parameter values corresponding to the desired inertia parameters.

$$f(\bar{\beta}) = \left((\frac{y_1^* - y_1}{y_1^*})^2 + (\frac{x_2^* - x_2}{x_2^*})^2 + (\frac{y_2^* - y_2}{y_2^*})^2 \right) * 10000 \qquad (8)$$

4.3 Results

In this work iterative optimization is used as a method of achieving the desired values of inertia parameters. Programming has been done in MATLAB®; the inbuilt optimization toolbox (*fmincon* routine) has been used. Target values for inertial parameters are taken from known geometry modeled using CAD software (Pro/E). Geometric constraints along with kinematic constraints are included for establishing the versatility of *geometry synthesis* procedure.

4.3.1 Unconstrained synthesis

Usually, simple shapes are assigned as nominal geometry of links which may not have the desired dynamic characteristics. Fig. 4 shows the initial geometry of a binary link, its evolution through intermediate stages and the final geometry satisfying the given requirement. The progress of the objective function with iteration is also shown. Edges in green color indicate those in the proximity of joint locations and in red color indicate their proximity with the boundary contour.No topological change was observed during shape evolution. Table 1 shows the result numerically. Average deviation of the values obtained is 0.09%. For the sake of brevity, numerical tables have been omitted for other examples.

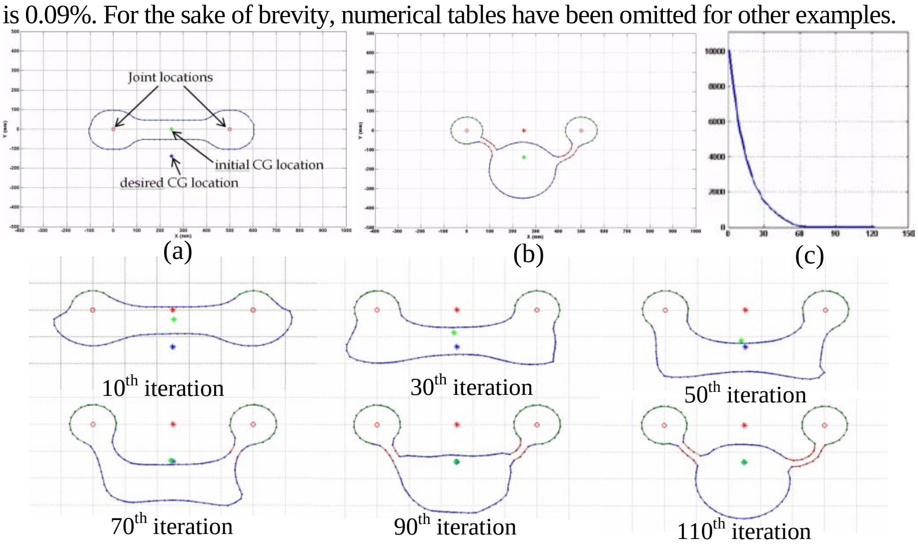

Figure 4: (a) Binary link initial geometry, (b) Final geometry after 134 iterations, (c) Objective function vs. Iteration, (d) Evolution of geometry

Table 1: Results for unconstrained geometry synthesis of binary link

Inertia parameter	Desired values	Initial Geometry	Final Geometry
Area (mm^2)	103896	95455.12	103784.05
CGx (mm)	250	250	249.94
CGy (mm)	-138	0	-138.15
I (mm^4)	12168194000	10254849228.21	12156025737.7

4.3.2 Constrained synthesis

The objective is to illustrate the geometric evolution in the presence of infeasible regions as constraints. Following are the different topological constraints used in the synthesis.

Fixed topology

Fig. 5 shows the result of geometry synthesis for a binary link including infeasible regions in the design space with fixed topology constraint. Distance value set for proximity checks between the boundary contours manifests in the minimum thickness of the component near the holes as in Fig. (5d). Average deviation in this case is 0.12%.

Minimal topology

Fig. 6 shows the result for a binary link with infeasible regions in design space and a minimal topology constraint; i.e. none of the existing holes are allowed to disappear. The initial domain has three infeasible regions. In this particular example the value of objective function did not converge; the average deviation in the final result is 6.71%.

Unconstraint topology

Fig. 7 shows the case where no topological constraint is imposed. This means that the number of holes can either decrease or increase but the procedure ensures that the evolving geometry is always a single component domain. The initial geometry has two holes containing infeasible areas. In this case objective function converged; the final geometry did not have any holes. The average deviation of the obtained values is 0.25%.

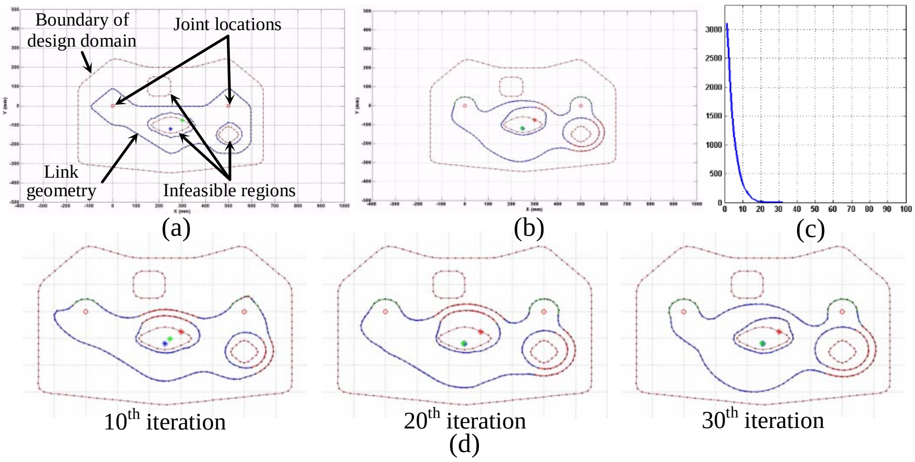

Figure 5: Geometry synthesis with fixed topology. (a) Initial geometry with infeasible regions, (b) Final geometry after 32 iterations, (c) Objective function vs. Iteration, (d) Evolution of geometry

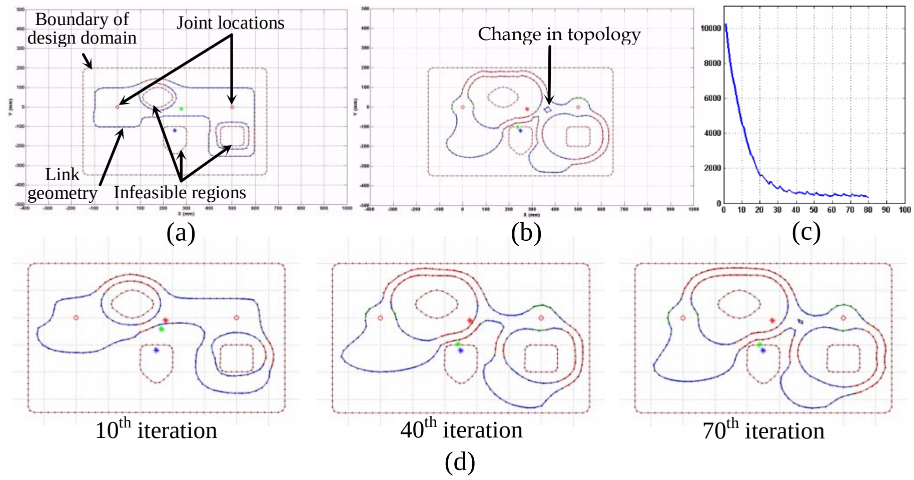

Figure 6: Geometry synthesis with persistent holes. (a) Initial geometry with infeasible regions, (b) Final geometry after 79 iterations, (c) Objective function vs. Iteration, (d) Evolution of geometry

4.3.3 Non-uniqueness of Solution

Wenoticed that shape changes significantly even when target parameters are almost reached. To explore this aspect further, optimization process was run with low tolerance large iterations. It can be observed in Fig.11 that beyond 100 iterations, error in the result is too small for any practical purpose. The final geometry that is obtained almost looks like the one from which the values of inertial parameters are taken from. Hence this methodology has the potential for shape reconstruction. It is intriguing that the intermediate geometries in Fig. 8(d) have shapes similar to that of a crank at the joint locations. These cases resemble the sector type cylindrical counter-weights used in [2] for balancing of linkages. The shapes are however significantly different. This establishes that a continuous *family of shapes* are available that satisfies the kinematic, geometric and inertial requirements.

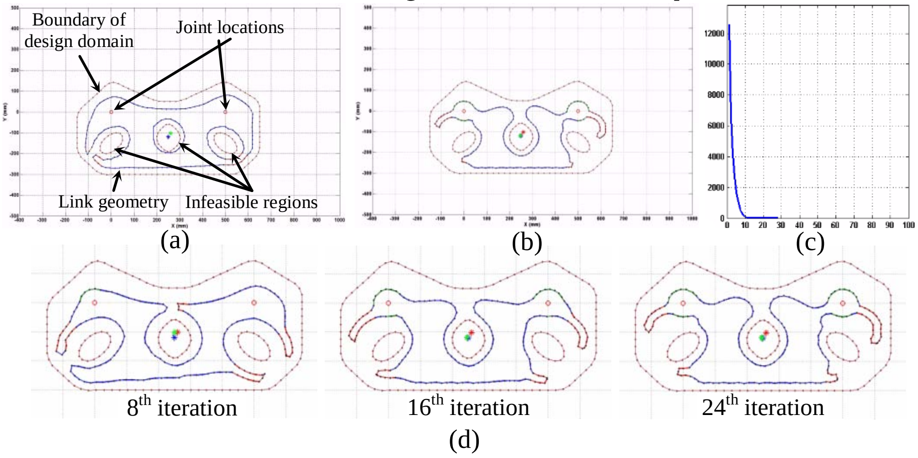

Figure 7: Geometry synthesis with unconstrained topology. (a) Initial geometry with infeasible regions, (b) Final geometry after 28 iterations, (c) Objective function vs. Iteration, (d) Evolution of geometry

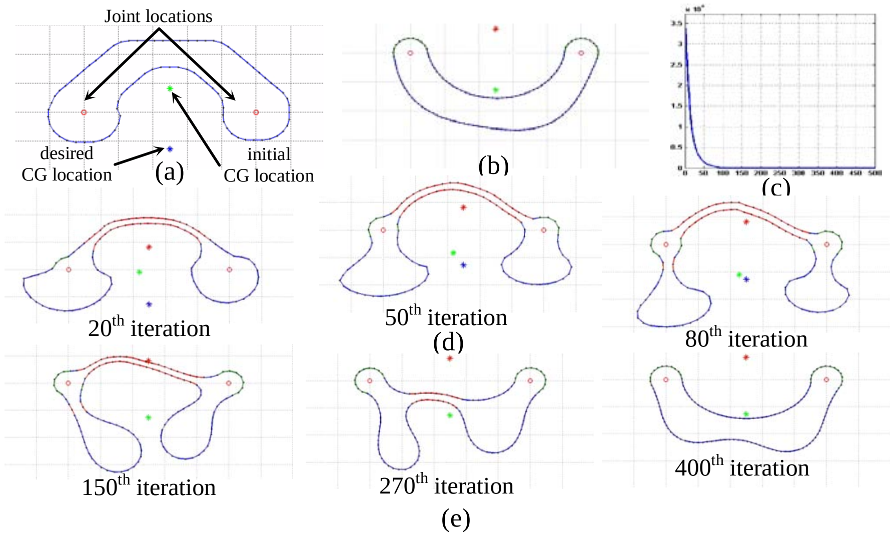

Figure 8: Geometry evolution beyond convergence: inertia-equivalent shape family. (a) Binary link initial geometry, (b) Final geometry after 500 iterations, (c) Objective

function vs. Iteration, (d) Intermediate geometries, (e) Multiple geometries with desired values of inertial parameters

5 Conclusions

A generic methodology was presented for the purpose of geometry synthesis with the ability of controlling the topological evolution of the domain represented as a set of closed polygons. Two *novel geometric transformations* have been introduced for controlling topology using local relation on edge segments in proximity. Shape modification is achieved by moving the vertices in normal direction to the boundary contour at each vertex. The proposed method of geometric evolution ensures that the resulting geometry be always a single continuum of material that can be multiply connected. The methodology admits easy imposition of different types of topological constraints. The method per say is generic enough for use in any problem requiring control on topology in its result. The work reported is believed to be the first of its kind in the field of mechanism design aiming towards explicit determination of geometry of each link for inertia requirement.

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