Analysis of Positional Error due to Joint Clearances in Four Bar Mechanism

H. P Jawale, H. T. Thorat

Abstract

Closed link mechanisms are widely used in machines and manipulators. The presence of clearance at the joints in mechanisms is inevitable. Joint clearance produces positional deviation along with other effects like vibration and wear. It is needed to study its effect as positional deviation caused due to it is unpredictable and random. It is needed to model possible highest positional error due to joint clearance and identify respective positions of mechanisms in workspace.This paper presents formulation of joint clearances in single-DOF planar mechanism. An orientation of coupler and follower is obtained for respective position of input link. Each clearance link is allowed to rotate fully about respective pivot centre. Shift in joint positions and mathematical expression of its motion is obtained. Formulation is implemented through soft computing. Highest positional deviation of coupler point with and without joint clearance is anticipated. It is observed that maximum error at any point on coupler is almost equal to sum of error due to individual joint clearances with almost equal contribution of all joint clearances in total error.

Keywords: Four bar mechanism formulation, clearance consideration, mechanism mapping, positional deviation, Maximum error.

1 Introduction

Closed link mechanisms are widely used in machines and manipulators. It can be thought of applying such manipulators for fast automations as these are economical than conventional open chain robots. Conventional planar mechanisms and other inversions generating fixed coupler curves are the primary phases of such applications. The closed chain mechanism can be synthesized as a manipulator by utilizing suitable drive to achieve desired motion of a coupler point as an end effecter. The presence of clearance at the joints in mechanisms is inevitable. Small suitable clearance is necessary to move a mechanism smoothly. On the other hand, adverse effects of joint clearance are noticed. Many researchers have modelled mechanism joint clearance as an additional link of length equal to half of the joint clearance to identify its influence on dynamic properties, positional deviation, vibrations and noise, accelerations and surface wear. [1-6] Position and orientation of the clearance vector is further governed by load on mechanisms, speed of operation, inertia forces and other dynamic factors. Positional deviation due to joint clearance is greater than cumulative effects of all other factors. It is needed to study its effect as positional deviation caused due to it is unpredictable and random. [7] Unlike the effects of link dimensional tolerances [8-11] and link stiffness calibration

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for effect of joint clearance is not possible as revealed in literature [12]. Thus compensation for this account is not possible at any of the stage. It is needed to model possible highest positional error due to joint clearance and identify respective positions of mechanisms in workspace. Prediction of positional error over entire workspace is important for evaluation of performance of mechanism for application as a manipulator.

Primary objective of work presented here is to model effect of joint clearance on positional deviation at a point on of coupler. Also analysis of the same for different path points is carried out.

1.1 Joint clearance model

Geometrically, joint clearance (δ) is the difference of radius of pin and the hole as in Fig. (1). Location of contact between pin and hole is controlled by forces at joint. Under this condition, clearance is modelled as a small link between nominal links of mechanism. Each clearance will add one degree of freedom (DOF) to the linkage. Thus, single DOF four-bar linkage behaves as a five DOF eight -bar linkage.

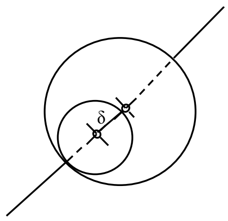

Figure 1: Radial clearance link at joint

2 Formulation for RRRR mechanism

The simplest and basic mechanism used for analysis of joint clearance is a coupler - crank four bar mechanism where crank can rotate fully about fixed centre. Input link is a crank and end effecter is on coupler as shown in Fig. (2). Link 2 is a fully rotatable crank and forms input link. Link 3 is a coupler link and link 4 forms output link. θ_1, θ_2, θ_3, θ_4 are the link angles made with horizontal fixed link axis. The formulation for four bar mechanism is carried out by considering initial link length such that the mechanism is a class - I mechanism. The proportions of link lengths used here are: $l_1 = 4 \times l_2$ and $l_3 = l_4 = (l_1 + l_2)/1.5$.

For a four link planar mechanism shown in Fig. (2), relation between link lengths in vector form is specified by applying Loop Closure Equation [13]

$$l_3 + l_4 \geq l_2 + l_1 \quad (1)$$

Displacement analysis of the mechanism for link angle gives

$$\theta_2 = (\alpha_1 + \beta_1) - \alpha_2 \quad (2)$$

$$\theta_3 = (\alpha_2 + \beta_2) + 180 \quad (3)$$

$$\alpha_1 = \cos^{-1} (l_2^2 - l_1^2 + r_1^2) / (2 \times l_2 \times r_1) \quad (4)$$

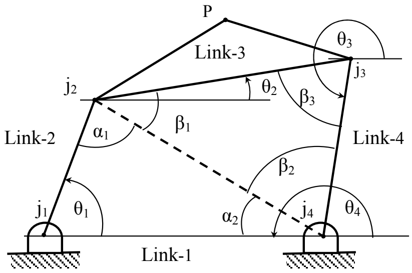

Figure 2: Four Bar RRRR mechanism

Where r_1 is the shortest distance between joint j_2 and j_4. Similarly α_2, β_1, β_2 can be found out. The formulation provides position of coupler point about joint j_1 for any input link angle.

3 Formulation of RRRR mechanism with clearance

Due to joint clearance, coupler point is shifted to P_1 from P as shown in Fig. (3) Each joint is shifted due to clearance link. Position Let λ_1, λ_2, λ_3, and λ_4 are angle made by clearance link about j_1, j_{21}, j_{31} and j_4 with fixed link axis and δ_1, δ_2, δ_3, and δ_4 are the clearance link lengths, the new position of joint j_1 is $j_{11}(x_{11}, y_{11})$ such that

$$x_{11} = x_1 + \delta_1 \cos \lambda_1, \; y_{11} = y_1 + \delta_1 \sin \lambda_1 \tag{5}$$

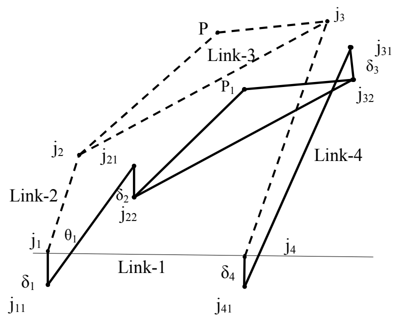

Figure 3: Shift in coupler position due to clearance link

For analysis, λ is considered to be varying from 0^0 to 360^0. Position of joint j_2 is shifted to $j_{21}(x_{21}, y_{21})$ due to clearance link δ_1. Similarly because of link δ_2 at joint j_2, position of j_{21} is shifted to $j_{22}(x_{22}, y_{22})$ such that

$$x_{21} = l_2 \cos \theta_1 + x_{11}, \; y_{21} = l_2 \sin \theta_1 + y_{11} \tag{6}$$
$$x_{22} = x_{21} + \delta_2 \cos \lambda_2, \; y_{22} = y_{21} + \delta_2 \sin \lambda_2 \tag{7}$$

Also similar considerations for remaining joints j_4 provides

$$x_{41} = x_4 + \delta_4 \cos \lambda_4,\ y_{41} = y_4 + \delta_4 \sin \lambda_4 \tag{8}$$

Where θ_1, θ_2, θ_3, and θ_4 are major link angle at joint j_{11}, j_{22}, j_{31}, and j_4 respectively, in counter-clockwise direction with respect to fixed link. Equation 5-8 provides effect of clearance on positions of joints. Links between joints j_{22}, j_{31}, j_{32} and j_{42} forms a four bar configuration with δ_3 as fully rotatable crank.
Thus for an input link angle, position of coupler point with and without joint clearance is obtained. The deviation in position is the resulting positional error. All values of error are expressed in terms of root of square of error in x and y direction.
It represents radius of an error circle for a coupler position. Numerical values of error are dimensionless entity indicating ratio with clearance link length in denominator. The formulation provided here is capable of considering effect of each individual link on coupler point position in workspace. Applying soft computing tool almost all possible combinations can be considered for analysis.

4 Steps for quantification of error

The objective of mathematical formulation presented here is to analyze effect of clearance on positional error. It is implemented in steps for prediction of error.
a) Effect of Single Clearance link is considered by rotating it about a joint through angle λ varying from 0^0 to 360^0. The positional deviation of end effecter in workspace is obtained so that remaining links are considered to have zero lengths.
b) Effect of clearance at all joints is analysed by rotating them through 0^0 to 360^0 in step of 10^0. Maximum positional error in all set of combinations of orientations of joint clearances is found for each combination in an iterative process.
c) Maximum Error in Workspace - Mechanism when moved through finite positions on coupler curve, presence of clearance gives error which will vary at different locations. Region of maximum error in workspace is to be identified for efficient operation of mechanism.
d) Shift in position of coupler point along coupler link l_3 from joint j_2 to j_3 positional deviation will vary. For analyzing effect of such variation coupler point is considered to move from zero to maximum length along coupler link.
e) Effect of variation in clearance link length as in practical situation is analysed by varying it at different joints. Sensitiveness of maximum error for such variation can be predicted with this formulation.
f) Nominal Error under force of gravity may be different than maximum positional error as its orientation is uncertain in absence of positive drive. The mechanism is acted by gravity factor during normal motion in workspace. Thus nominal positional error is actual positional error computed by considering clearance link at $\lambda = 270^0$ with respect to ground link in counter clockwise direction.

5 Results and Discussion

Analysis of mechanism with above methodology is carried out. Results are as under
1) Effect of a clearance link rotation about a joint in mechanism is shown in Fig. (4) Any of the clearance links when rotated about the joint, error varies from 0^0 to 360^0 of rotation counter-clockwise with respect to fixed link axis. This pattern is observed

for all positions of clearance link δ_1 and δ_2, with positional error ε_1, ε_2 are exactly same and both curves overlap. Similar results are obtained for δ_4 as ε_4 overlapping half the curve for ε_1, ε_2 and increasing to higher value near 270^0 Clearance link δ_3 contributes symmetrically as ε_3. Further it is observed that highest value is noted whenever input link aligns with fixed link axis i.e. angle of rotation of crank is 0^0 or 180^0.

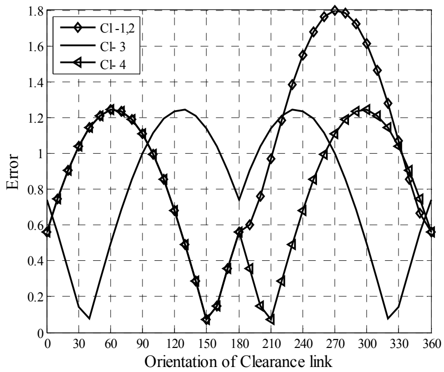

Figure 4: Effect of clearance individual link on error

Table-1 shows typical positions of all individual clearance links λ_1 to λ_4 at máximum error. Orientations of nominal links for maximum error condition are found to be identical for identical clearance length δ. ($\theta_1 = 60^0$, $\theta_2 = 60^0$, $\theta_3 = 120^0$, $\theta_4 = 110^0$, $\delta = 0.01$ and nominal link lengths are $l_1 = 8000$, $l_2 = 2000$, $l_3 = 6666.667$, $l_4 = 6666.667$)

Table 1: Position of individual clearance links for maximum error

Parameter	Clearance link under consideration			
	C link - 1	C link -2	C link -3	C link -4
Clearance link orientation, λ	60	60	120	110
Error, ε	1.798	1.798	1.242	1.242
Total Error, ε_T				6.08
Individual link Angles for all links considered simultaneously	$\lambda_1 = 280$	$\lambda_2 = 280$	$\lambda_3 = 60$	$\lambda_4 = 130$
Maximum Error ε_{ALL}				5.897

2) Maximum error is found to vary throughout workspace by considering all clearance links simultaneously as well as complete crank rotation as shown in Fig. (5) Corresponding values are 5.876.

3) Maximum error at various points on coupler found to increase from joint j_2. At joint j_2, error is exactly equal to sum of clearance link at previous two joints. Higher error is observed for crank near fixed link axis.(angle $= 0^0$ and 180^0) As position of coupler point is moved towards j_3 value increases being maximum at end point of coupler as shown in Fig. (6) The variation can be mapped as in Fig. (7) Contribution of each individual link in total error at various positions of end effecter is shown in Table -2.

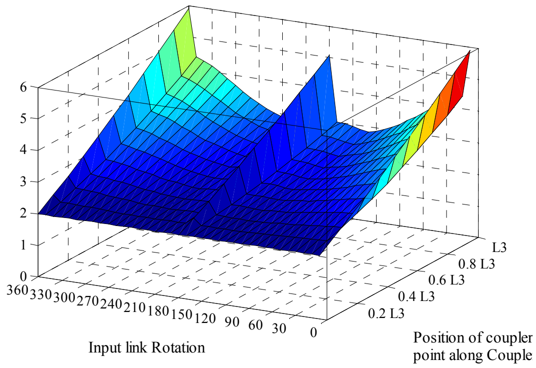

Figure 5: Effect of all clearance links on error.

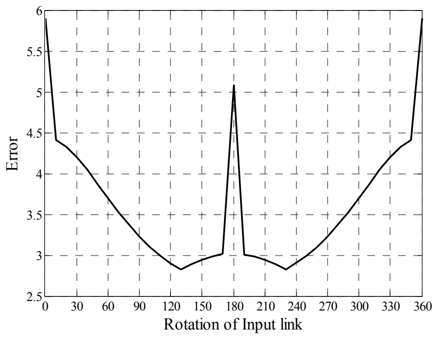

Figure 6: Variation of error for crank rotation

Table 2: Contribution of Each Clearance link at various coupler positions

Position along coupler link	ε_1	ε_2	ε_3	ε_4	Total Error ε_T	ε_{ALL}
0	1	1	0	0	2	2
0.1×l3	1.064	1.064	0.124	0.124	2.376	2.331
0.2×l3	1.131	1.131	0.248	0.248	2.758	2.677
0.3×l3	1.203	1.203	0.373	0.373	3.152	3.04
0.4×l3	1.279	1.279	0.497	0.497	3.552	3.419
0.5×l3	1.358	1.358	0.621	0.621	3.958	3.807
0.6×l3	1.44	1.44	0.745	0.745	4.37	4.201
0.7×l3	1.524	1.524	0.869	0.87	4.787	4.618
0.8×l3	1.609	1.609	0.994	0.994	5.206	5.041
0.9×l3	1.703	1.703	1.118	1.118	5.642	5.467
l3	1.798	1.798	1.242	1.242	6.08	5.897

4) When clearance at all joints is considered simultaneously, the value of error ε_{ALL} is marginally smaller ($\leq 3\%$) than total sum of deviation due to individual clearance links ε_T as seen from values in table-1 for extreme point on coupler link. Table-2 shows similar observation for various positions from zero length to full length on coupler. Graphical representation is as shown in Fig. (8)

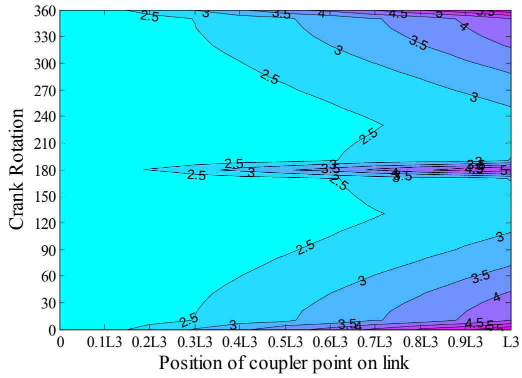

Figure 7: Error zones for mechanism operation.

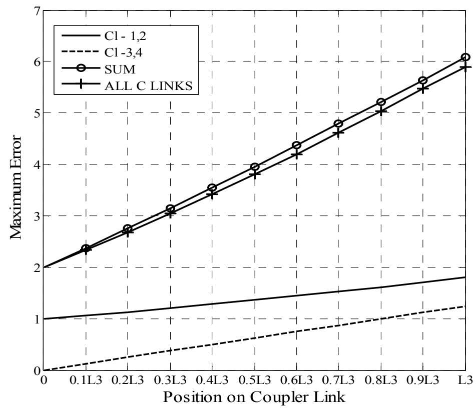

Figure 8: Contribution of all links in maximum error

5) Variation of clearance link length has very small effect on maximum error as indicated in Table -3 and represented graphically in Fig (9). Contribution of clearance link 1 and 2 is same and lines are overlapped over one another, whereas link 3 shows reduction in error over range of variation and link 4 has a constant effect.

Table 3: Effect of variation in clearance link length

Clearance link length	ε_1	ε_2	ε_3	ε_4	Total Error ε_T	ε_{ALL}
0.0001	1.798	1.798	1.469	1.242	6.307	6.116
0.001	1.798	1.798	1.239	1.242	6.077	5.894
0.01	1.798	1.798	1.242	1.242	6.08	5.897
0.1	1.798	1.798	1.242	1.242	6.08	5.897
1	1.799	1.799	1.242	1.242	6.082	5.897
10	1.799	1.799	1.242	1.242	6.082	5.903
100	1.807	1.807	1.238	1.238	6.09	5.961

6) Orientations of all clearance links are beyond control as those cannot be driven positively. The position of clearance link remains fixed without rotation under gravity and will have variation in error due to crank rotation. The maximum error of order

3.495 is obtained in region as shown in Fig (10). The curve shows higher values at positions parallel to fixed link.

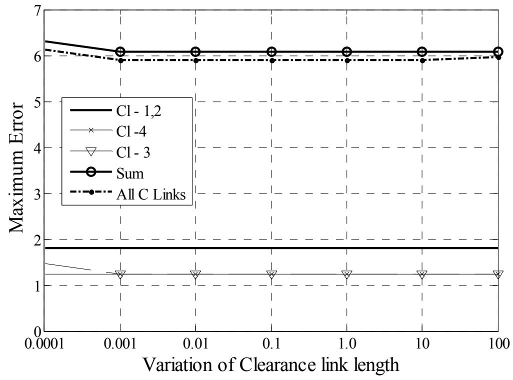

Figure 9: Effect of Clearance link length variation on Error

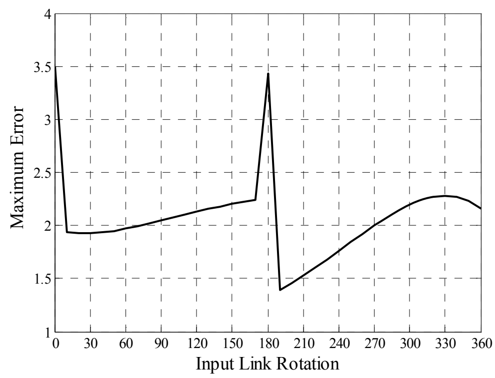

Figure 10: Maximum Error under gravity

6 Conclusions

1) Contribution of all clearance links is approximately equal in positional error in workspace.
2) Maximum error increases linearly with respect to position of coupler point on link, as location shifts away from joint with input link.
3) Maximum positional error at any point on a coupler link is marginally small (≤ 3.0%) than the sum of contribution of individual clearance link in workspace of mechanism.
4) Above conclusions holds valid for any value of clearance from 0.0001 to 100 units of nominal link length.
5) If clearance effect is considered taking into account only gravity maximum positional error is smaller than the sum of clearance link lengths.

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