Control and Stability Analysis of a Walking Knee-less Biped with Torso

Abdul Jaleel, Tripuraneni Varun, Arun D. Mahindrakar

Abstract

Seminal works in biped walking such as [1] had assumed favorable initial conditions on a limit cycle to start with. A more practical approach would be to start the biped from a static resting position. This paper proposes a simplified and straightforward approach for taking a biped robot from an initial resting position to a stable walking limit cycle. The biped model selected for the study is the 'knee-less biped with torso'. The problem is tackled in two stages - *gait initiation* from rest followed by convergence to *stable walking*. Walking is divided into various sub-phases depending on the state of the biped and simple state feedback control laws are proposed for each phase. The individual control laws are mathematically accrued into a single control law valid throughout the walking phase. Simulation results validating the approach are presented.

Keywords: Knee-less biped, Walking robots, limit cycle

1 Introduction

The study of human locomotion and attempt at its imitation by biped robots have been an active area of research for many years. The motivation behind this interest are many. It can be seen that legged locomotion is often the most ideal mode for mobility over rough terrains. In fact only half of the earth's landmass is accessible to wheeled and tracked vehicles. The advantage of legged locomotion in this regard can be attributed to isolated footholds which optimize support and traction. Yet another advantage is that the payload (body) can move smoothly irrespective of the roughness in terrain as legs decouple the path of the body from the path of the feet. A legged system can also step over obstacles. Further, research in this area can help us gain better understanding of human and animal locomotion. Motivated by these reasons, a large amount of work has gone into the study of legged robots. A detailed description of the initial research and various milestones in this regard can be seen in [2].

Legged locomotion is studied under various modes such as walking, running, skipping etc. In this work we focus on walking. It has been proved that bipeds are capable of walking down slopes without any control inputs. This is called passive walking and can be seen in [3], [4]. In [5], the author has shown the existence of a class of walking machines that settle into a steady gait quite comparable to human walking when

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started with shallow slope, without active control or energy input. In [6], the authors extend the stability analysis of the simplest walker [7] to obtain improved and accurate analysis of the simplest walker. Active walking with the help of controllers is of more practical interest and a wide variety of control laws have been proposed for the same. In [3], the authors argue that mimicking the passive gait via control will have its advantages. A portion of control strategies proposed in literature for biped locomotion depend on trajectory tracking. This is done using continuous PID controller [8], computed torque control [9] etc. Other control approaches which do not rely on trajectory tracking include energy tracking control laws [10], control of angular momentum [11] and intuitive control strategies [12]. Other works have attempted the use of foot actuation via impulsive foot action [13].

In this work we aim at taking a simple biped model into a stable walking limit cycle. While most of the previous work on biped walking have assumed the biped to be already in a favourable initial condition within the limit cycle, we attempt to initiate the biped into walking from a resting initial position - a more realistic scenario. This gait initiation and the succeeding walking cycle are achieved using a set of feedback control laws.

2 Mathematical Model

The primary step in the study of legged robots is the selection of an appropriate mathematical model to represent the system. A large variety of models have been proposed in literature with varying degrees-of-freedom and complexity. They differ in the presence of knees, torso, limbs and other features[1, 14]. For selecting an appropriate model for study, a compromise has to be made between accuracy of the model and its simplicity. The most simple model seen in literature representing a biped is the compass gait model in [3]. The simple addition of a torso to this model resulted in a challenging yet tractable model called the 'knee-less biped with torso'.

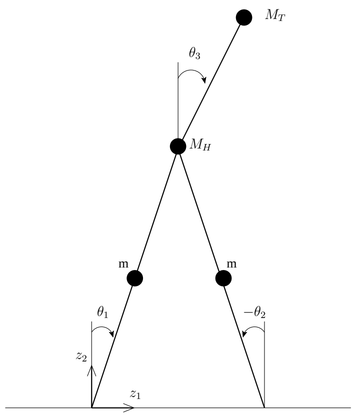


Figure 1: Knee-less biped with torso

This biped model consists of two legs and a torso as shown in Fig. 1. The angle θ_1 corresponds to the stance leg, θ_2 the swing leg and θ_3 the torso. The legs are taken to be symmetric with length r. Mass m of each leg is assumed to be lumped at a distance of $r/2$. The hip is assumed to be a point mass M_H. The torso Center-of-Mass (COM) with mass M_T is taken at a distance l from the hip. The hip contains two actuators, one for each leg. It is to be noted that for the above model to be able to walk without scratching the surface, a foldable or retractable tip is assumed as in [10]. Keeping the mass of this tip negligible, this provision can be neglected in the theoretical study of the biped walking.

Walking consists of two alternating phases - the swing phase and the impact phase. Hence a hybrid model is used to describe the dynamics associated with it.

The dynamic model of the swing phase for the walker is obtained using Euler-Lagrange equations given by $\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i} = \tau_i,\ i = 1, 2, 3$ where the Lagrangian L is the difference between kinetic and potential energies of the system and τ is the vector of external torques. Note that $\tau_3 = 0$. For the walker in swing phase, $q_s = (\theta_1, \theta_2, \theta_3)$ and the dynamics are

$$M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + G_s(q_s) = B_s(q_s)u \tag{1}$$

where,

$$M_s = \begin{bmatrix} r^2(\frac{5}{4}m + M_H + M_T) & -\frac{1}{2}mr^2c_{12} & M_T rlc_{13} \\ -\frac{1}{2}mr^2c_{12} & \frac{1}{4}mr^2 & 0 \\ M_T rlc_{13} & 0 & M_T l^2 \end{bmatrix}; B_s = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$C_s = \begin{bmatrix} 0 & -\frac{1}{2}mr^2\dot{\theta}_2 c_{12} & M_T rl\dot{\theta}_3 s_{13} \\ \frac{1}{2}mr^2\dot{\theta}_1 s_{12} & 0 & 0 \\ -M_T rl\dot{\theta}_1 s_{13} & 0 & 0 \end{bmatrix}; G_s = \begin{bmatrix} d \\ \frac{1}{2}gmr\sin\theta_2 \\ -gM_T l\sin\theta_3 \end{bmatrix}.$$

$c_{12} \triangleq \cos(\theta_1 - \theta_2), c_{13} \triangleq \cos(\theta_1 - \theta_3), s_{12} \triangleq \sin(\theta_1 - \theta_2), s_{13} \triangleq \cos(\theta_1 - \theta_3), d \triangleq -\frac{1}{2}g(2M_H + 3m + 2M_T)r\sin\theta_1$. With $x = (q_s, \dot{q}_s)$, (1) can be written in state-space form as

$$\dot{x} \quad = \quad f_s(x) + g_s(x)u. \tag{2}$$

During the impact phase the swing leg comes in contact with the ground. The stance leg is assumed to leave the ground simultaneously with this impact. Hence the impact phase is instantaneous and can be modeled as a discrete map, in other words a mapping from states 'before' impact to 'states' after impact. This impact model is derived using the principle of conservation of angular momentum as explained below.

Unlike the swing phase where we only required three states (as stance leg tip remains fixed over a step) the impact phase requires five states. Thus we augment z_1, z_2 to q_s to obtain $q_e = (\theta_1, \theta_2, \theta_3, z_1, z_2)$. The equations thus obtained are

$$M_e(q_e)\ddot{q}_e + C_e(q_e, \dot{q}_e)\dot{q}_e + G_e(q_e) = B_e(q_e)u + \delta F_{ext}. \tag{3}$$

The vector δF_{ext} represent the external impulse forces acting on the body during contact. Integrating (3) over the duration of impact we can obtain

$$M_e(q_e)(\dot{q}_e^+ - \dot{q}_e^-) = F_{ext}. \tag{4}$$

where $\dot{q}_e^+$ is velocity after impact, $\dot{q}_e^-$ is velocity before impact and $F_{ext} = \int_{t^-}^{t+} \delta F_{ext}$. Here t^- is the time just before impact and t^+ just after. The co-ordinates of the end of swing leg is given by $\gamma(q_e) = \begin{bmatrix} z_1 + r\sin\theta_1 - r\sin\theta_2 \\ z_2 + r\cos\theta_1 - r\cos\theta_2 \end{bmatrix}$. Further let F_T, F_N be the forces (in the Cartesian space) applied at the tip due to impact and define $F = [F_T \; F_N]^\top$. The transformation from Cartesian space to joint space gives $F^\top\dot{\gamma} = F_{ext}^\top\dot{q}_e$. But $\dot{\gamma} = \frac{\partial\gamma}{\partial q_e}\dot{q}_e = E\dot{q}_e$ where $E = \frac{\partial\gamma}{\partial q_e}$. Substituting in $\dot{\gamma}$ and rearranging $F^\top E - F_{ext}^\top = 0$ yields $F_{ext} = E^\top\,[F_T \;\; F_N]^\top$. Assuming that the leg neither rebounds nor slips on impact, leads to

$$\frac{d}{dt}\gamma(q_e) = (\frac{\partial\gamma}{\partial q_e})\dot{q}_e^+ = E\dot{q}_e^+ = 0. \tag{5}$$

Combining (4) and (5) we obtain

$$\begin{bmatrix} M_e & -E^\top \\ E & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_e^+ \\ F \end{bmatrix} = \begin{bmatrix} M_e\dot{q}_e^- \\ 0 \end{bmatrix}. \tag{6}$$

Also it can be seen that the legs swap their role on impact. Thus θ_1 and θ_2 are interchanged whereas θ_3 remains same on impact. Hence $\theta_1^+ = \theta_2^-; \theta_2^+ = \theta_1^-; \theta_3^+ = \theta_3^-$ which along with $\dot{q}_e^+$ from (6) forms the discrete map representing the impact model and is denoted by

$$x^+ = \triangle(x^-). \tag{7}$$

where $\triangle$ returns the states after impact (x^+) as a function of states before impact (x^-).

The overall hybrid model of the biped walker is obtained by combining (2) and (7). For this first we define the walking surface S as $S = \{(q_s, \dot{q}_s) : \theta_1 + \theta_2 = 0\}$. The overall hybrid model of the three-link walker can then be written as

$$\begin{array}{rcll} \dot{x} & = & f_s(x) + g_s(x)u & x^-(t) \notin S \\ x^+ & = & \triangle(x^-), & x^-(t) \in S. \end{array} \tag{8}$$

3 Proposed Control Law

The control objective is to initiate walking in the biped in a visually appealing gait, and continue the walk until a stable limit cycle is reached. The verification of stability of the limit cycle will is carried out using Poincaré maps, which is omitted for the sake of brevity. The objective of walking from rest was achieved in two stages. The first phase aimed at kick-starting the motion by moving the torso front and hence make the biped fall forward. The next phase aimed at manoeuvring this falling biped into a stable walking limit cycle behaviour. This phase is further divided into three sub-phases depending on the angle of the stance leg. Specific objectives are assigned for each sub-phase and state feedback control laws are used to achieve the same.

3.1 Phase I - gait initiation

To initiate walking, the biped's center-of-gravity have to be shifted ahead. This is achieved by rotating the torso forward using the stance-leg actuator. The aim is only to reach a required angular position, and not to stabilize around it. Therefore, a negative feedback control using the angular velocity of the torso would achieve the same result, keeping the angular velocity within a specified limit. The control law used is $\tau_1 = k_{I,st}(\dot{\theta}_3 + 1.5) + 10e^{10\theta_3}$. The first term $k_{I,st}(\dot{\theta}_3 + 1.5)$ contains the main feedback control, which brings the torso to a terminal angular velocity of $1.5 \mathrm{~rad/s}$ in the clockwise direction. The second term $10e^{10\theta_3}$ only increases the rate at which the system moves towards it's feedback aim.

3.2 Phase II - stable walking

Phase II is a sequences of sub-phases which begins the moment the swing leg detaches from the surface, and ends when the step is made. Every step the biped makes is one iteration of phase II. The second phase consists of three sub-phases: 1) From the start of Phase II till the stance leg becomes vertical, and is about to fall forward, 2)from the end of sub-phase I till the swing foot reaches a desired position above the surface, 3) from the end of sub-phase II till the swing foot makes contact with the surface.

3.2.1 Sub-phase I

The local objective of this sub-phase is to: a)Bring the swing leg forward towards the stance leg, b) to reduce the difference between $|\theta_1|$ and $|\theta_2|$, in order to make the gait resemble that of a human, c) to reduce the torso angular velocity as much as possible. The stance leg control law $\tau_{st,sub-phase\ I} = k_{1,st}\left(-m\left(\dot{\theta}_1 + (\pi/2)\right)^2 + \dot{\theta}_3\right)$ is aimed at taking the angular velocity of the stance leg to a required value, in addition to the feedback control on the angular position of the torso. The first term $-k_{1,st}m\left(\dot{\theta}_1 + (\pi/2)\right)^2$ is the negative feedback control which settles the angular velocity of the stance leg to a value of $\pi/2 \mathrm{~rad/s}$. The second term $k_{1,st}\dot{\theta}_3$ attempts at reducing the angular velocity of the torso. The swing leg control is aimed at reducing the difference between the angular positions of the swing and stance legs. In addition to moving the swing leg forward, this feedback control makes the gait resemble the human walking gait. The feedback control $\tau_{sw,sub-phase\ I} = k_{1,sw}\left(-\theta_1 - \theta_2\right)$ is a state feedback controller, with an adjusted coefficient. The first sub-phase ends at the moment the stance leg crosses the vertical.

3.2.2 Sub-phase II

This sub-phase is crucial in increasing the overall velocity of the biped, and in monitoring the step-length in the current and subsequent steps. The local objective of this sub-phase is to: a) Continue the motion of the stance and swing legs, in their respective directions, b) increase the clearance gap between the swing foot and the ground, in order to detect the position at which the step fall can be initiated (Phase III), c) bring

the torso angle to a desirable value.
The stance leg control law $\tau_{st,sub-phase\,II} = k_{2,st}\,(\theta_3 + (\pi/6))$ is aimed at bringing the torso angle to a desirable value, which here is $\pi/6$. The desirable value given in the feedback control here is $\pi/6$. Note that the torso might never reach the given desired value, but the feedback control will achieve the target of stabilizing the value of the torso angle to a certain small range.

The swing leg control law is aimed at the first and second objectives. The first objective is met in a way similar to the swing leg control law of Phase I. The second objective is dependent on the step length to be taken by the biped. A clearance of $5\ \mathrm{cm}$ is taken as the goal of the feedback control, and the resulting law is $\tau_{sw,sub-phase\,II} = k_{2,sw}\left[-\theta_1 - \theta_2 + \frac{l}{\theta_0}\right]$ where, θ_0 is the initial angular position of the stance leg, just before the current step began. The presence of θ_0 ensures that the step length taken in the current step is approximately equal to the previous step. This prevents the step length from increasing drastically, which lowers the biped, in which case the subsequent step might not be able to overcome the sudden loss in Potential Energy. The second sub-phase ends when the stance leg reaches a certain given angle, which is a simple function of θ_0. This is also to ensure that the step-length does not change too much.

3.2.3 Sub-phase III

This is the sub-phase in which the impact occurs. The local objective of this sub-phase is to: a) Manoeuvre the swing leg so that the foot attains a near vertical angle of approach just before impact, b) bring the torso angle to the desirable value, i.e. $\pi/6$. The stance leg control law $\tau_{st,sub-phase\,III} = k_{3,st}\,[\theta_3 + (\pi/6)]$ is again aimed at the torso angle.

The swing leg control law is aimed at the first objective. The expression in the feedback control
$\tau_{sw,sub-phase\,III} = k_{3,sw}\left[\cos(\theta_1)\,\dot{\theta}_1 - \cos(\theta_2)\,\dot{\theta}_2 - p\,(\theta_2 - (\pi/15))\right]$ is the derivative of the horizontal velocity of the swing foot.

3.3 Mathematical accruement

The requirement is to integrate the three control laws into a single continuous function. With the help of 'switching' functions, the overall control law can be written as $\tau_i = f_1 C_1 + f_2 C_2 + f_3 C_3$ where C_1, C_2, C_3 correspond to the control laws of the first, second and third sub-phases respectively and f_1, f_2, f_3 are the switching functions selected such that they have a constant value (preferably 1) during the corresponding sub-phase and is zero elsewhere. Hence during sub-phase 1, $\tau_i = 1 * C_1 + 0 * C_2 + 0 * C_3 = C_1$ and so on. The natural choice for such a switching function would be rectangular pulses. But this renders the system discontinuous, hence continuous functions which approximate the rectangular pulse were sought. The tan hyperbolic function has a behaviour which can be manipulated to obtain this approximation. Two appropriately constructed $\tanh(\,\cdot\,)$ functions can be added to obtain a function which is very close to a rectangular pulse. The sub-phases of the biped were defined based on the value of the stance angle, θ_1. Hence tan hyperbolic functions of θ_1 were used to obtain the switching functions for the biped.

4 Simulation Results

The equations (8) were solved to obtain the continuous evolution of the states. The integration was stopped on detecting an 'impact' of the swing leg with ground, denoting the end of the current step.'Impact' was detected by the ODE solver using its built-in 'events' function which kept checking for $\theta_1 = \theta_2$ with every integration step. On detecting an impact the integration was terminated and the final states were passed on to a discrete mapping function which represents the change in states due to impact. The equations obtained for the impact phase were used for calculating the new states after impact. These new states were then passed back to the ODE solver, which accepts them as the new initial conditions and restarts integration as before. This whole cycle is repeated until a 'fall' is detected or the biped walks a prescribed number of steps or if a specified time limit is reached. From the initial rest condition, Phase I of the control law was successful in pushing the torso forward. The leg angles (θ_1 and θ_2) remain constant at the initial values throughout the phase whereas θ_3 decreases (as clockwise is considered negative). The plot of angles during gait initiation are shown in Fig. 2(a). After successful gait initiation, the phase II of the controller becomes active and tries to initiate the biped towards a walking behaviour. The biped converging to a limit cycle walking was successfully simulated. The limit cycle was obtained when the biped reached the state $x = (-.1157, .1157, -.4565, -1.3261, -0.3949, 2.1681)$. This convergence can be seen from the plot of θ_1 vs θ_2 as shown in Fig. 2(b). The stick animation of the biped converging to the limit cycle and then walking without fail for any prescribed number of steps is shown in Fig. 2(c).

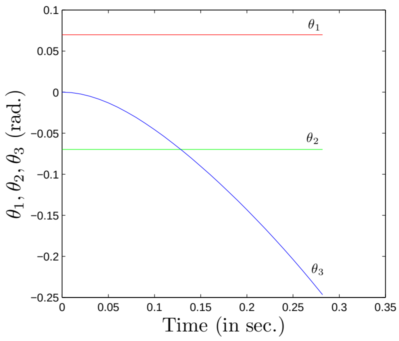
5 Conclusion

The knee-less biped model was successfully taken into a stable limit cycle corresponding to a walking gait using simple control laws. The problem was simplified by dividing the walking cycle into two main stages - gait initiation and stable walking. Stable walking was further tackled in three sub-phases. Individual objectives were identified for each sub-phase and feedback control used to achieve them. The walking so obtained was further analyzed for stability using Poincaré Return maps and found to be stable. The robot was also found to be stable against slipping.

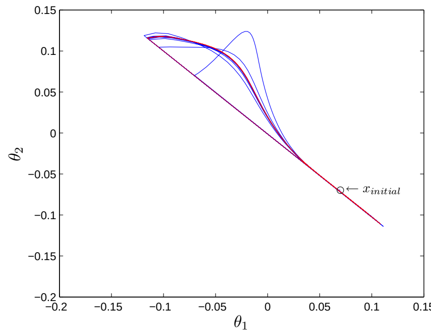
References

[1] J. Grizzle, G. Abba, and F. Plestan, "Asymptotically stable walking for biped robots: Analysis via systems with impulse effects," *IEEE Transactions on Automatic Control*, vol. 46, no. 1, pp. 51–64, 2001.

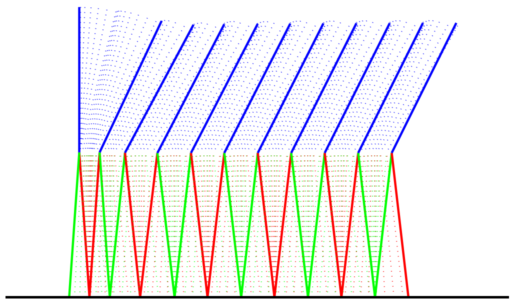
[2] M. Raibert, "Legged robots," *Communications of the ACM*, vol. 29, no. 6, pp. 499–514, 1986.



(a) Link angles during gait initiation



(b) The closed trajectory corresponding to limit cycle is shown in red. The sudden jump in the plot corresponds to the impact



(c) The stick animation frames of the biped taking 10 steps from rest. The gait initiation by the torso can be easily observed

Figure 2: Gait initiation and walking

[3] B. Thuilot, A. Goswami, and B. Espiau, “Bifurcation and chaos in a simple passive bipedal gait,” *in proceedings of the IEEE International Conference on Robotics and Automation*, vol. 1, pp. 792–798, 1997.

[4] R. Van Der Linde, “Active leg compliance for passive walking,” *in proceedings of the IEEE International Conference on Robotics and Automation*, vol. 3, pp. 2339–2344, 1998.

[5] T. McGeer, “Passive dynamic walking ,” *International Journal of Robotics Research*, vol. 9, no. 2, pp. 62–82, 1990.

[6] S. L. Das and A. Chatterjee, “An alternative stability analysis technique for the simplest walker,” *Nonlinear Dynamics*, vol. 28, pp. 273–284, 2002.

[7] M. Garcia, A. Chatterjee, A. Ruina, and M. Coleman, “The simplest walking model: stability, complexity, and scaling,” *Journal of Biomechanical Engineering*, vol. 120, pp. 281–289, April 1998.

[8] J. Furusho and M. Masubuchi, “Control of a dynamical biped locomotion system for steady walking,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 108, pp. 111–118, 1986.

[9] J. Park and K. Kim, “Biped robot walking using gravity-compensated inverted pendulum mode and computed torque control,” *in proceedings of the IEEE International Conference on Robotics and Automation*, vol. 4, pp. 3528–3533, 1998.

[10] A. Goswami, B. Espiau, and A. Keramane, “Limit cycles and their stability in a passive bipedal gait,” *in proceedings of the IEEE International Conference on Robotics and Automation*, vol. 1, pp. 246–251, 1996.

[11] A. Sano and J. Furusho, “Realization of natural dynamic walking using the angular momentum information,” *in proceedings of the IEEE International Conference on Robotics and Automation*, pp. 1476–1481, 1990.

[12] J. Pratt and G. Pratt, “Intuitive control of a planar bipedal walking robot,” *in proceedings of the IEEE International Conference on Robotics and Automation*, vol. 3, pp. 2014–2021, 1998.

[13] J. Choi and J. Grizzle, “Feedback control of an underactuated planar bipedal robot with impulsive foot action,” *Robotica*, vol. 23, no. 05, pp. 567–580, 2005.

[14] C. Chevallereau, E. Westervelt, and J. Grizzle, “Asymptotic stabilization of a five-link, four-actuator, planar bipedal runner,” *in proceedings of the 43rd IEEE Conference on Decision and Control*, vol. 1, pp. 303–310.