Design Optimization Of Toggle Mechanism By Mathematical Model

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Abstract

This paper investigates the optimization of toggle mechanisms by energy approach. Circuit breaker mechanism is the classical example of toggle mechanisms. High degree of reliability is required for circuit breaker mechanism to ensure safety of the electrical circuit and human life. Extra available (reserved) energy in the toggle mechanisms can be used during life of the product to ensure high degree of reliability. Total available energy in the mechanism is used to find the dynamic response of spring operated mechanism from the known value of spring stiffness called forward dynamics. The equation of motion, second order non linear differential equation and special form of Lagrange equation, used to analyze the dynamic response of the spring type operating mechanism. Equation of motion is solved in MATLAB gives the motion of the system as a function of time. Result of closing operation is comparable with those obtained numerically by Nastran Working Model as well as by experiment. These results of equation of motion used to find the frictional energy loss in the mechanism. Next step is to find out the extra available energy called energy margin in the toggle mechanisms.

Keywords: Toggle Mechanism, Lagrange, Dynamic, Virtual work

1 Introduction

A circuit breaker can be manually opened and closed, as well as automatically opened to protect conductors or equipment from damage caused by excessive temperature from over current in the event of a short circuit. For open operation of a circuit breaker, important consideration is breaking time. Generally breaking time should be in the order of milliseconds to avoid arching. Prolonged arcing overheats and melts the moving and fixed electrical contacts. For close operation of circuit breaker, moving contact must snap the fixed contact with some contact pressure to avoid arcing. The rapid making or breaking of a circuit breaker demands the operating mechanism to be tough, durable and safe to withstand incoming power supply so that in situations of system breakdown, they function as required [1].

This paper investigates the optimization of available energy in the toggle mechanism to optimize the energy margin. The energy margin in the mechanism can be found out by energy available, energy required with energy losses in the mechanism. Energy losses due to friction can be found out by complete dynamic analysis of the breaker mechanism. The dynamic response of circuit breakers is generally analyzed and simulated using computer aided design packages. The limitation with computer aided design packages are, whenever there is a dimensional change, the 3-D model needs to be rebuilt and then simulate that model to get the

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results. That is not the case with mathematical model; one can simulate the system by changing the parameters [2].

The dynamic response of circuit breakers can be analyzed using multi-body dynamics [3]. In multi-body dynamics, the number of differential equations (Lagrange Equation) increases exponentially with increasing complication. Not only does the derivation process become tedious, the computing time also increases significantly when solving these equations numerically. But the use of equation of motion (simplified form of Lagrange equation) reduces complexity for the single degree of freedom multi-body system which can be formulated by single equation so the computing time automatically reduces.

2 Circuit Breaker Operating Mechanism

In the close operation, as illustrated in Fig. (1), link5 (L5) is locked by the tripping latch and hence remains stationary. Link1 (R1) is the driving link, and link4 (R4) generates the output motion. At the same time, the contact spring (R8) for close operation is compressed. In open operation as the tripping latch is turned on, the tripping spring rotates link5 (L5) to generate the output motion. At the same time, link4 (R4) is rotated clockwise from the effect of the contact spring (R8). In undesirable circumstances, the tripping and closing springs in this circuit breaker, which are constantly in the ready mode, can be manually overridden to perform the sequence open–return–close–open–return in an instant [1-2].

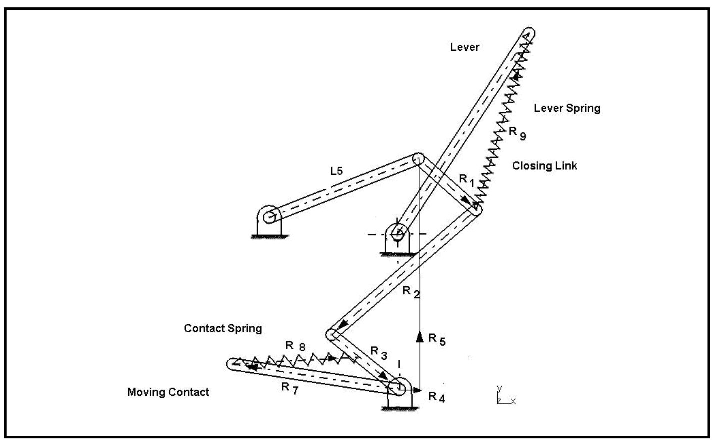

Figure 1: Vector- loop diagram of Operating Mechanism.

3 Equation of Motion

If all rigid bodies constitute a mechanical system having single degree-of-freedom, their motions can be represented by a second-order non-linear differential equation [2-6] as

$$I(\theta)\ddot{\theta}(t)+C(\theta)\dot{\theta}^2(t)=M(\theta)\text{..}(1)$$

This is also known as the generalized equation of motion. θ is the angular displacement of the input link, $I(\theta)$ is the generalized inertia, $C(\theta)=(1/2)(d[I(\theta)]/d\theta)$ is the generalized damping and $M(\theta)$ is the generalized moment.

If a mechanism is formed by n number of links, then $I(\theta)$ and $C(\theta)$ in Eq. (1) can be represented by

$$I(\theta) = \sum_{i=1}^{n} \left[m_i ({}^j h^2_{gix} + {}^j h^2_{giy}) + I_i {}^j h^2_i \right] \qquad (2)$$

$$C(\theta) = \sum_{i=1}^{n} \left[m_i ({}^j h_{gix} {}^j h'_{gix} + {}^j h_{giy} {}^j h'_{giy}) + I_i {}^j h_i {}^j h'_i \right] \qquad (3)$$

where m_i is the mass of link i, I_i is the mass moment of inertia of link i, ${}^j h_i$ is the kinematic coefficient of link i relative to the input link (link j) and ${}^j h_{gix}$ and ${}^j h_{giy}$ are the kinematic coefficients of the centre of gravity of link i in the X and the Y axis respectively relative to the input link (link j). The sign (‘) represents the derivative of the variable when using the equation of motion to analyze the dynamic response of a spring-type operating mechanism circuit breaker, the kinematic coefficients of the members and their centers of gravity must first be derived to obtain the coefficients in the equation of motion. The equation can then be solved by numerical iteration after applying the initial conditions and the results from positional analysis.

The next step is to calculate the inertia forces by D’Alembert’s principle and torque required to operate the mechanism by using matrix method or by virtual work method.

4 Friction in the dynamic analysis

In dynamic analysis of systems, frictional force depends on the resultant reactive forces between the two mating surfaces. Resultant reactive forces depend not only on the static loading but also on the forces developed by the motion of the mechanism. However, the motion of the mechanism is derivable only when the friction force is known [6, 10].

The collapse of the toggle mechanism in the presence of Coulomb friction method utilizing both equations of motion and the kinetostatic analysis is outlined to solve frictional torque and it is concluded that the performance of the mechanism under the influence of Coulomb friction depends highly upon the input energy to the system [6].

The complex nonlinear characteristic of the Coulomb friction, especially in the dynamic mode at a joint, makes the analysis cumbersome. This type of friction produces some form of energy dissipation, which is a function of torque, which is in turn a function of instantaneous dynamic load, as illustrated in Fig. (2). Coulomb friction depends on the joint reactions, and its contribution to the generalized force cannot be determined until the equations of motion are solved. This reasoning suggests a procedure whereby the equations of motion are solved using the energy method over a small step, with Coulomb friction ignored. Therefore, the assumption is that no friction exists at time zero. Then the joint reactions, including friction, are found from a kinetostatic analysis by solving for two nonlinear equations. Find the torque due to this friction and substitute this in the equation of motion as an external torque for the next time interval.

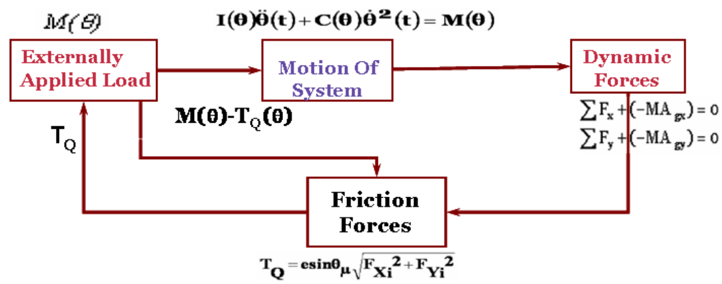

Figure 2: Flow Chart for the equation of motion with the effect of friction.

5 Energy analysis and Efficiency of circuit breaker

The efficiency of the circuit breaker is defined as the ratio of sum of Kinetic energy loss in the mechanism, frictional losses in the mechanism and energy stored in the opening spring to the energy stored in the closing springs in the charged condition. Following are the values of energy stored in various springs. These values are calculated using the formula,

$E = 0.5 \times k \times x^2$..(4)

Where,

E= Energy stored in spring, k = Spring constant & x = Extension in spring.

1. Energy released by Closing Spring = E_1
2. Energy stored in Opening Spring = E_2
3. Energy stored in Pole Spring = E_3
4. K.E. loss in the mechanism = $E_4 = E_3 + E_6$
5. Frictional energy loss in the mechanism = E_5
6. Energy loss due to impact = E_6
7. Energy margin = $E_7 = E_1 - (E_4 + E_5 + E_2)$

Efficiency of Breaker $= \left[\left(E_2 + E_3 + E_7\right)/E_1\right]$...........................(5)

6 Estimation of Losses during closing operation

In order to improve efficiency, it is essential to find out if the energy stored in the closing spring is actually essential for the proper functioning of the circuit breaker. If the energy stored in the closing spring can be reduced without affecting the performance of the circuit breaker, then the efficiency can be improved or by improving the energy margin means minimizing frictional energy losses and improving the conversion of kinetic energy loss of mechanism to changing the pole spring.

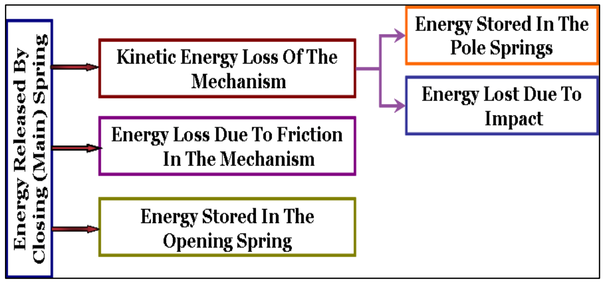

Figure 3: Flow of Energy in Circuit Breaker Mechanism

6.1 Kinetic Energy of the entire mechanism

During closing of the circuit, the velocity of each link can be formulated by differentiating the position of the centre of gravity (CG) of the link with respect to time. The total kinetic energy of each link is then the sum of the kinetic energies of the translation and the rotation of the link.

$$T_2 = \frac{1}{2} m_2 V_2^2 + \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2}(m_2 R_{g2}^2 + I_2)\dot{\theta}_2^2 \quad \text{.................................(6)}$$

Similar equations for remaining links,

Where,
T_i = Kinetic Energy, m_i= Mass of Link (Kg)
V_i= Linear velocity of link (m/s)
θ_i = Angular Velocity of link (rad/sec.)

6.2 Energy lost due to friction in various joints

Frictional losses between the joints were calculated as follows. Frictional loss in each time step is given by

$$E_{loss} = \Gamma \times \theta_t \quad \text{..(7)}$$

Where,
E_{loss} =Energy lost in one times step
Γ =Frictional torque acting on joint.
θ_t = Rotation of the joint in the time step.

6.3 Energy lost due to impact

Energy loss due to deformation is the dominant mechanism of energy dissipation in collisions between compact solid bodies with aspect ratios near unity

6.3.1 Frictional losses from the two surfaces sliding against one another

Frictional losses are manifested from the surfaces of the two bodies sliding against each other during the contact period [7-8].

6.3.2 Production of sound from induced vibrations in the surrounding air

If the collision produces symmetrical spherical waves from a single source, the average rate of energy flow through a spherical surface of radius 'r' is $W_{sound} = \left(4\pi r^2 P^2 / 2\rho_o c\right)$..(8)

Where,
P = Pressure, c = wave speed in air, and ρ = density of undisturbed air.

6.3.3 Energy transformed into internal vibrations of one or both bodies

The amount of energy lost to vibrations in the bodies has been studied by Rayleigh, investigated the amount of the initial kinetic energy lost to vibrations when an elastic sphere collides with a half space [9-10].

Rayleigh, $\left(E_{Vib}/E_{in}\right) = 1/50\left(V_0/C_0\right)$..(9)

Where,

V_o is the normal relative speed at impact and C_o is the wave propagation speed.

6.3.4 Energy losses due to material deformation

Since all other means of energy dissipation are negligible for the collisions of moving and fixed poles, the losses due to material deformation must be almost entirely responsible for the energy loss during impact [9].

Energy Loss Due To Impacts
$W_c = 0.5\times\left(\mathrm{I}_1\omega_1^2\right) - 0.5\times\left(\mathrm{I}_2\omega_2^2\right)$..(10)

7 Results and Discussion

After deriving the equations of motion of a spring-type operating mechanism for a circuit breaker in close operations, they are then solved by substituting the dimensions, masses, centers of gravity and moment of inertia. Mass, centre of gravity and moment of inertia are computed by constructing a solid model of the individual machine members using the computer-aided designing software. Given the initial position of the mechanism, and that the initial velocity is zero, Eq. (1), a second-order non-linear differential equation, can easily be solved using the fourth-order Runge–Kutta method [MATLAB].

Results for the close operation are illustrated in Fig. (4) and the comparison with experimental results which show good agreement. The predicted energy margin in the close operation is 18%.

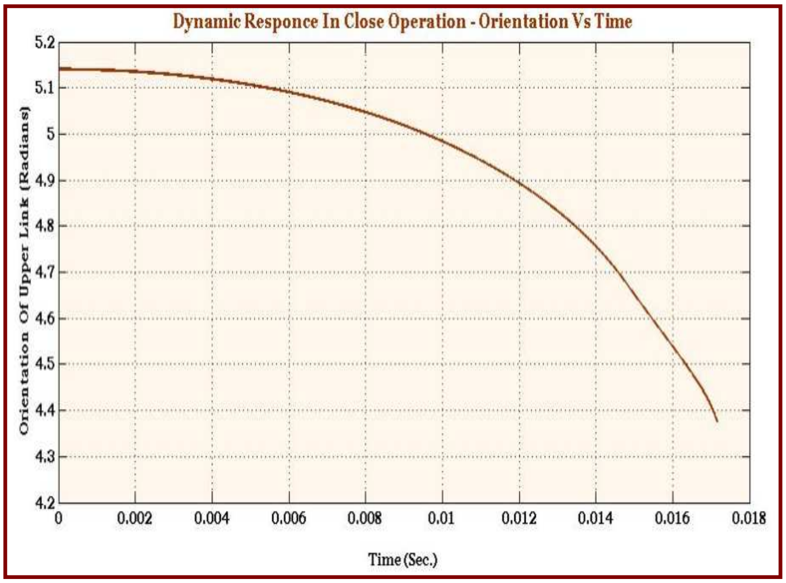

Figure 4: Dynamic response in close operation

Reliability of circuit breaker depends on contact pressure between the electrical contacts. Wear and tear of electrical contacts, due to arcing, may reduce life of circuit breaker. This excess energy (Energy Margin) in the mechanism can be used to enhance the life of circuit breaker.

Table 1: Summary of Energy Distribution

Sr.No	Energy Distribution	Energy (J)
1	Energy Released By Closing Spring(Main Spring)	35.97
2	Energy Stored In Contact Springs	6.08
3	Total Kinetic Energy Of Mechanism	10.16
4	Total Energy Loss In Impact Of Moving Fingers	2.01
5	Total Energy Loss In Impact Of Toggle Pin	0.31
6	Total Energy Loss In Friction	3.30
7	Energy Stored In Opening Springs	15.68
8	Energy Margin	6.53

Table 2: Efficiency of Breaker and Energy Margin

Sr.No	Description	%
1	Efficiency Of Circuit breaker	60.47
2	Energy Margin	18.15

8 Conclusion

In case of circuit breakers, due to long static periods of inactivity the system needs to have high degree of reliability. One of the function of a circuit breaker is to close (ON) the mechanism positively with some contact force between the electrical contacts (contact depression).If contact depression is not sufficient then there will be arcing between moving and fixed electrical contacts as the magnetic forces tries to lift the contacts, so heat is generated which erodes the contact tip thereby reducing the life of the breaker to a great extent. Always it is preferable to close the breaker with extra available energy with some contact depression. The equations of motion derived in this paper can accurately predict the durations of the close operation of a spring-type operating mechanism as well as be used to compute the dynamic response of the moving contact. Furthermore, the dynamic response analyzed herein is also applicable to other spring-type operated mechanisms, such as hinges etc. During dynamic analysis minimum torque required to operate the mechanism can be found out from that data optimization of main closing spring. Therefore it is stressed here that the energy margin approach is very efficient to improve the efficiency of circuit breaker (toggle) mechanism.

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